Learning Objectives:

1) Calculate the final vel. of rocket.

2) Calculate periods, energies, angular momenta of planets.

3) Explain in your own words how to transfer linear momentum from one marble to another.
LEARNING OBJECTIVES: (cont’d)

4) Explain in your own words what gravity assist is.

5) Work out specific examples of gravity assists.

6) Explain in your own words how can a spacecraft launched from Earth reach Saturn faster than on fuel alone.
LINEAR MOMENTUM

\[ \vec{P} = m \vec{v} \]
LINEAR MOMENTUM
CONSERVATION:
\[ \vec{P}_i = \vec{P}_f \]
\[ M d\vec{v} + dm \vec{v}_{rel} = 0 \]
ROCKET EQUATIONS:

$$\vec{V}_f - \vec{V}_i = - \vec{V}_{rel} \ln \frac{M_i}{M_f}$$

$$T = \left( \frac{dM}{dt} \right)_{\text{rel}} \overline{\text{THRUST}}$$

WHERE $$\frac{dM}{dt} = - \frac{dm}{dt}$$
Example:

\[ V_{rel} = 3 \text{km/s}, \quad \frac{dM}{dt} = 200 \text{ kg/s} \]

\[ \therefore \text{Thrust is:} \]

\[ T = \frac{dM}{dt} \cdot V_{rel} = (3 \text{km/s})(200 \text{kg/s}) = 600,000 \text{N} \]

\[ \max \frac{T}{g} = \frac{600,000 \text{N}}{9.81 \text{m/s}^2} \approx 61,000 \text{kg} \approx 61 \text{ton} \]
EXAMPLE: (cont'd)

\[ M_i = 3000 \text{ kg}, \quad 0.2000 \text{ kg in fuel} \]

\[
\frac{dM}{dt} = 0.003 \text{ kg/s} - v_i = 0, \quad v_{rel} = 3 \text{ km/s}
\]

\[ v_f - v_i = -(-3 \text{ km/s}) \ln \frac{3000 \text{ kg}}{10000 \text{ kg}} = 3.3 \text{ km/s} \]

\[ t_{burnout} = \frac{2000 \text{ kg}}{0.003 \text{ kg/s}} = 185 \text{ hours} \]
Newton's Law of Gravitation

\[ F = G \frac{m_1 m_2}{r^2} \]
NEWTON'S 2nd LAW:
\[ \vec{F} = m \vec{a} \]
ORBITS: NEWTON'S GRAV = NEWTON'S 2nd

\[
\frac{G m_1 m_2}{r^2} = m_1 a_1
\]

\[
\frac{G m_1 m_2}{r^2} = m_2 a_2
\]
KEPLER'S 3 LAWS:

1) ALL PLANETS ORBIT SUN IN ELLIPSES WITH SUN BEING AT ONE FOCUS.

2) VARIABLE SPEEDS, FASTEST @ PERIHELION, SLOWEST @ APHELION

3) $P^2 = a^3$
   
   - Period (semi-major axis (distance from center of ellipse to...)}
Total Energy (circ. orbit)

\[ E = \frac{1}{2} m v^2 + \left( -\frac{G m M}{r^2} \right) \]

Kinetic Energy

Potential Energy
Angular Momentum

\[ \mathbf{L} = m \, \mathbf{v} \times \mathbf{r} \]
**Linear Momentum Cons.:** \( m_{1i} \, \vec{V}_{1i} + M_{2i} \, \vec{V}_{2i} = m_{1f} \, \vec{V}_{1f} + M_{2f} \, \vec{V}_{2f} \)

**Kinetic Energy Cons.:** \( \frac{1}{2} m_{1i} \, v_{1i}^2 + \frac{1}{2} M_{2i} \, v_{2i}^2 = \frac{1}{2} m_{1f} \, v_{1f}^2 + \frac{1}{2} M_{2f} \, v_{2f}^2 \)

**Energy Cons.:** \( \frac{1}{2} m_{1i} \, v_{1i}^2 + \frac{1}{2} M_{2i} \, v_{2i}^2 = \frac{1}{2} m_{1f} \, v_{1f}^2 + \frac{1}{2} M_{2f} \, v_{2f}^2 \)
\[ V_{16} = \frac{(1 - m/M)V_{1i} + 2V_{2i}}{1 - m/M} \]

\[ \text{if } M \gg m \Rightarrow V_{16} = V_{1i} + 2V_{2i} \]

\[ P_{16} = m(V_{1i} + 2V_{2i}) \]
Gravity Assist (very simplified!)

\[ (M_J \gg m) \]