Activity 1: solve for size and speed of a drop.

5. In order not to be breakable, the surface tension of a drop should be greater than the pressure difference between top and bottom of the drop.

It implies that \( \sigma a \sim \Delta P a^2 \).

And \( \Delta P \) is equal to \( \frac{\rho v^2}{2} \).

\[ \sigma a \sim \rho_{\text{air}} v^2 a^2 \]

At the terminal velocity, drag force is equal to gravity.

\[ D = \rho_{\text{air}} v^2 a^2 \sim mg \sim \rho_{\text{w}} a^3 g \]

\[ \frac{\sigma}{a} \cdot a^2 = \sigma a \sim \rho_{\text{w}} a^3 g \]

\[ a \sim \sqrt{\frac{\sigma}{\rho_{\text{w}} g}} \]
Useful units for calculations

Remember: dyne = 1 g cm/s² (force in cm-g-s units) = 10⁻⁵ Newtons. Using cgs units is easiest for calculations with small insects, as the values are often order-one.

- Density of air \( \rho = 10^{-3} \text{ g/cm}^3 \)
- Density of water \( \rho_w = 1 \text{ g/cm}^3 \)
- Gravity \( g \approx 1000 \text{ cm/s}^2 \)
- Surface tension \( \sigma = 70 \text{ dynes/cm} \)
Answer 1: Raindrops heavier and faster than mosquitoes

- raindrop radius ~ 1 - 4 mm
- raindrop mass ~ 2 - 50 mosquitoes
- raindrop speed ~ 5 - 9 m/s >> mosquito speed (1 m/s)

*Clements, A. The sources of energy for flight in mosquitoes.

**Sayile, D. & Hayhoe, H. The potential effect of drop size on...
Activity 2: What is frequency of impacts for a flying mosquito?

This is a problem of mass conservation. Rain intensity $I$ is given in the units of cm/hour. We must convert this unit into a number of drops that falls on top of the mosquito per second.

This problem is related to the the first problem in “flying circus of physics”, which asks is wetter to run or walk through the rain. Here we recognize that mosquitoes fly so slowly that impacts on the mosquito’s frontal area are negligible compared to that on top.

So consider only drops falling atop the mosquito where plan-view area of wings and legs is $A_m$. This area is given by considering all drops that impact or even graze the legs (See diagram on next page where an area is sketched out that is one drop radius wider than legs and body). Students should estimate this area $A_m$ to 1 significant digit.

First convert $I$ to cm per second. Then, every second, we consider the volume of drops that fall to fill a volume that is $A_m$ wide and $I$ tall. We can convert this into a mass of fluid falling per second using the density of water $\rho$ used in activity 1. Lastly, we can find frequency of drops $f$ by remembering each drop has a fixed volume $m$ calculated from activity 1.
Activity 2 solution: Frequency of impacts

impact area \( A_m \sim 30-40 \text{ mm}^2 \).

rain intensity \( I \sim 50 \text{ mm/hr} \)

frequency of impacts: once every 25 sec

\[
f = \frac{\rho IA_m}{m_{drop}} \sim \frac{1}{25} \text{ hits/s}
\]

Direct impact (red) zone composes 1/4 total area.

Glancing Impacts are 3X more likely.
Activity 3: What is force of raindrop after glancing blow

This is a problem of conservation of angular momentum.

The
Activity 3: Glancing Blow

Torque applied by the drop: \( r \times F = I \alpha \)

Geometry: \( I = m_2 R^2/2 \approx 4 \times 10^{-5} \) g cm\(^2\)
\( R = 1 \) mm

\( \rightarrow F \sim 3.5 \) dynes \( \sim 2 \) mosquito weights

\( \alpha \sim 10^4 \) rad/s\(^2\)

\( \tau \sim 10^{-2} \) s
Direct Hit

Inelastic collision—mosquito and drop move as one mass. Mosquito pushed many body lengths before released.
Insect Mimics

Styrofoam “Mosquitoes”

$m_2 \sim 0.4 - 4 \text{ mg}$

$m \sim 2 \text{ mg}$

Free body impacts
Activity 4: What is the final speed of raindrop-cum-mosquito after impact?

Consider conservation of linear momentum. In particular consider the momentum before and after impact.
Mosquitoes are so lightweight, raindrops loose very little momentum (2-17%) upon impact.
Small drops slow down more

**Drizzle**
Small Drop ($m_1 / m_2 = 9$)
⇒ large deformation

**Rain**
Large Drop ($m_1 / m_2 = 100$)
⇒ slight deformation

$R_1 = 1.53 \text{ mm}$
$R_2 = 2.75 \text{ mm}$
Collisions with body of high inertia

Terminal velocity raindrop on a solid surface: \( F \sim \frac{m_1 u_1}{\tau} \approx 5 \times 10^4 \) dynes

which is \( 10^4 \) times the weight of a mosquito.
Larger drops apply more force

Position

Velocity

Small drop

Large drop

$\#g's = 65$

$\#g's = 95$
Mosquitoes Accelerate 50-300 G

\[ F = 2 \text{ dynes} \times 300 \text{ G} = 600 \text{ dynes} \approx 0.61 \text{ gf} \]

Max force: 4000 dynes (flight still possible) – 10,000 dynes (death)