

BLOSSOMS MODULE FLAW OF AVERAGES

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R: Hello and welcome to our Blossoms module on the flaws of averages. My name is Rhonda Jordan and I'm a graduate student here at the Massachusetts Institute of Technology in Cambridge, Massachusetts, USA.

We begin our module with two illustrations of some limitations of averages, which helped motivate us to make this module. Our first example is as follows:

On Thursday, I performed a modern dance, on campus, in this white dress. Then, Friday night, I went to MIT's elegant grad gala in my favorite black dress. For Saturday night, I was going dancing with some friends and I decided to wear something that was the average color of the two dresses. So what did I end up wearing? This. Which has neither the elegance of the black dress, nor the simplicity of the white dress. My friend Dan, who is also a graduate student at MIT, has a quick story about a day he had recently that will help illustrate what we mean by the flaws of averages.

D: Thanks Rhonda! Hi! My name is Dan Livengood. And like Rhonda I'm also a graduate student here at MIT.

For our second example, I'd like to tell you a quick story about a day that I had recently. On that day, I spent four hours in the morning here at MIT. Then, in the afternoon, I spent four hours across the river in downtown Boston. So, if I were to stand in my average location over those eight hours, where would I be? Yep, I'd be standing on the Charles River. At least there was a sailboat for me stand on! Hello again! I'm Dan.

R: And I'm Rhonda. As we said earlier, we are both graduate students at the Massachusetts Institute of Technology in Cambridge, MA, USA. Today we're here to talk with you about the flaws of averages.

D: Now before we start we do want to be clear that we're not trying to say that averages are bad. In many situations, averages provide a very good, single descriptive number of a situation. For example, what's the average height of all the students in your class?

R: Exactly! Now, I'm sure you can come up with a number of other situations where the average number is a good description.

D: The main point of this Blossoms module is simply to point out a few pitfalls that could arise if you're not attentive to detail when you're using and interpreting averages.

R: So let's get started and consider a case where the average may not always be a good description of the real situation!

D: Sounds good.

R: So Dan, let's imagine that you're at the edge of a river that you want to cross. But, there's a sign. The sign says, "Average river depth one meter." Now, given this sign, would you cross the river?

D: Hummm. Well, if I saw the sign I would think, "I'm 1.8 meters tall. I'm taller than the average river depth. So I should be able to walk across and keep my head above the water all the way on average, right?"

R: That's true, if this is your situation. If this is the water and this is the riverbed, it goes straight across at one meter, then you'll definitely be able to keep your head above the water.

D: OK. Well I can think of another example where I'd be able to keep my head above the water. So again, if this is your water, and this is your new riverbed shape, if for the first half of the river it's 0.5 meters deep, and then for the second half of the river it's 1.5 meters deep, on average it's still 1 meter deep. With only 1.5 meters as the maximum depth, I'd keep my head above the water.

R: All right Dan. But here's the kicker. Let's say this is your riverbed. OK? And you start at zero meters here and the riverbed goes all the way down to two meters. The average depth is still one meter, but in this two meter area, if you can't swim, I think you'd have a hard time keeping your head above the water, Dan.

D: You know, you're right, Rhonda. So class, what other riverbed shapes can you come up with? And for each one that you come up with, am I going to be able to keep my head above the water if I just decide to walk across the river, especially if I can't swim?
Hummm.

R: We'll let you guys discuss this with your teacher and your classmates.
We'll see you later!

R: Welcome back! I hope you had a nice discussion with your teacher about how the average is not always a good description of the actual situation. That is our flaw of averages number one.

D: And moving right along, our flaw of averages number two is that the function of the average is not necessarily the same as the average of the function.

R: Dan, what do you mean by that?

D: Well Rhonda, it just so happens that I've got a nice little example to help explain this flaw of averages. And I think you're going to like it.

R: Sounds good.

D: So I have two plates of cookies.

R: I'm SO hungry! Can I have some?

D: Sure. I'll let you have one plate and then I'll have the other.

R: OK. But like I said, I'm really hungry so can I have the plate that will fill me up the most?

D: Sure! What I'll tell you is that underneath A is a plate with two circular cookies that have an average diameter of 7 cm. And underneath B there's a plate with two circular cookies that have an average diameter of 8 cm. So which plate would you like to have?

R: Oh Dan, that's easy. I want plate B. The average diameter of those cookies is larger.

D: OK. Let's look at the two cookies that you've just chosen. Now would you like to see the two cookies on plate A?

R: Sure, why not? Let's see.

D: Well, here you go. On plate A you'll see that we have one VERY large cookie and one very small cookie.

R: Well, wait. The area on this plate is larger than the area of cookies on this plate. That's not fair, Dan.

D: Well, that's the exactly the point of this flaw of averages. So class, do you understand why Rhonda ended up with a plate of smaller area of cookies even though she picked the plate with the larger diameter on average? Talk this over with your teacher and your class and we'll see you when you get back.

R: Hey Dan, what's that over there?

D: What's what?

R: That, right there...

D: I don't know.

D: Welcome back! So far we've discussed two flaws of averages. The first one was that the average is not always a good description of an actual situation. The second flaw of averages is that the function of the average is not always the same as the average of the function.

R: Now the third flaw that we'd like to introduce is that the average depends on your perspective.

D: All right. So what do you mean the average depends on your perspective?

R: OK. Take me for instance Dan. So I'm a dance teacher and I teach two dance classes, one beginners' class and one advanced class. My beginners' class has 45 dancers, but my advanced class has five dancers. So if we took the average of these two numbers, we need to add $45 + 5$ and then divide by 2. We get 25. So Dan do you see how that's 25?

D: I do see how that's 25.

R: OK. But what if we actually interview all of the students and ask each one of them how many dancers are in your class including yourself? If we take the average of all of their responses, do you think we'd get the same answer?

D: Well, yeah, I don't know. I don't see why it would be any different than the 25 that we just calculated.

R: Dan, I actually did ask all of my students this question. And these are their 50 responses: forty-five 45s, and five 5s. So I took the average of all of these numbers and I got 41.

D: Wait a minute! 41? But why is that so different than the 25 we calculated earlier?

R: Dan, that's exactly and precisely the key lesson of this segment. So class, why is it that when we calculate the average from my perspective you get 25 dancers. But if you calculate the average from the dancers' perspective you get 41? Talk this over with your teacher and your classmates and we'll see you when we come back.

R: Hello again! We hope you all have enjoyed our module on the flaws of averages. Let's review what we've done today. Our first flaw of averages is that the average is not always a good description of the actual situation.

D: Our second flaw of averages is that the function of the average is not always the same as the average of the function.

R: Now our last flaw is that the average depends on your perspective.

D: Now, these three flaws that we've discussed today are not necessarily the only flaws of averages, and we encourage you to look and find other examples where averages can be misleading.

R: But we wanted you all to have a sense of a few of the pitfalls that might arise if you are not attentive to details when calculating and interpreting averages.

D: As we leave you today, we have one last example of that first flaw of averages—that the average is not always a good description of the actual situation.

R: So in our free time Dan and I love to perform. I'm a tap dancer,

D: And I sing.

R: And so we're going to put the two together for you.

(Song and dance.)

R: Now if we took the average number of taps that I just tapped,

D: And take the average note that I just sang,

R: You'll be listening to the following:

(Another example.)

D: Now hopefully you agree with us that the first version was much more interesting than the average!

R: So now we're going to say our good-byes and leave you with a longer version of our song and dance. Happy studying!

D: Goodbye!

(Song and dance.)

D: Hello there! Welcome to our flaws of averages Blossoms Module's teacher's segment. Rhonda and I had a lot of fun putting this together, and we're glad that you're interested in discussing this with your students. So, we wanted to go over a few of the things that we had in mind with the module. To begin with, we mention this in the video and we just want to stress that we're not trying to say that averages are bad! Averages do have their place. At one point we discussed the average height of the students in your classroom. That's just one example of how the averages could be very helpful. But we wanted to go over a few of the pitfalls that you could run into if you're not careful with using averages. So we'll go over each of the flaws of averages that we discussed. And don't forget you may have other flaws of averages that you've come up with, or that you think of, that you could go over with your students. So feel free to adjust and adapt this in any way that you find is helpful to explain this to your students. But again, we want to go over some of the things that we had in mind for you to go over in between the different segments. So we'll start off with our flaw of averages number one, that the average is not always a good description of the actual situation. We'll be right back.

D: And we're back to discuss our first flaw of averages, that the average value may not always be a good description of the actual situation. So what did we have in mind with this one?

You may have noticed that there are a few examples along the way to illustrate this point. The two of our opening segments, myself on the river, Rhonda in her dresses, both were designed to help illustrate this point. And we wanted to go over some of the things we felt that you could talk over with your students after this first segment. One of the concepts that you could talk about is that the average value may not be an actual outcome of the situation. So what do I mean by that?

Let's take this coin here. If I flip a head, I'm going to give you a dollar. If I flip a tail, I'm going to give you zero dollars. So if I were to flip this coin many times, on average I'd be giving you 50 cents. But wait a minute, 50 cents is not one of the outcomes. The outcomes are either a dollar or zero dollars. So that's conceptually what we were trying to get at with this one.

Another concept that you could discuss with your students is that depending on the situation, the average may be exactly the same, but the distribution may be different. So if you have a more advanced class, one of the things we had in mind was you could discuss the normal distribution, exponential distribution, whatever kinds of distributions you want to look at. For example think of the crossing of the river. We had the flat line, we had the sloped line, we had the sort of double table top line. Both of those were essentially three different distributions that all have the same average value. So those are the kinds of things that we were thinking you could discuss with your students after this first flaw of averages. You may have some others and feel free to check out the website which will have some more examples of what we had in mind for these. But for now, that's it for this part. And now we'll go on to the second flaw of averages, that the function of the average is not necessarily the same as the average of the function.

D: And now we can talk about our second flaw of averages, that the function of the average is not always the same as the average of the function. So if you've looked at our other segments for the module you probably remember the example with the cookies. So we had

plate A with two cookies and plate B with two cookies, where the average diameter for plate A of the two cookies was 7 cm and the average diameter of the two cookies on plate B was 8 cm. And as Rhonda helped us beautifully illustrate, intuitively you would think that the plate B set of cookies would be bigger in terms of area since the diameter is bigger. But as you saw, the cookies on plate A were actually bigger when we uncovered them. So that was just a nice way to illustrate this point that the function of the average is not always the same as the average of the function. Now we've left the math details to a file on the website. So if you're interested in discussing this in further detail with your students, feel free to go look at that page. We've also got another example about wind turbines, and I'm sure you can think of many other examples of functions where again the function of the average is different than the average of the function. So we'll now move on to our last flaw of averages, that the average value depends on your perspective.

D: Our last flaw of averages to discuss with you here in the teacher's segment is that the average depends on your perspective. So from the main video our example was Rhonda's dance classes. And we've got a number of other examples on the website that you can look at. I wanted to bring to your attention one of the examples, though, about Lake Woebegone. Now Lake Woebegone is a fictional town from a radio program here in the US, and one of the key phrases about Lake Woebegone is that "all of the children are above average." Now this may sound like an impossible statement, but again it all depends on your point of view. If the average that you're discussing is a national average, it could be possible that Lake Woebegone's children could all be above average. We have more details on the website so feel free to go look at those. We hope you've enjoyed working with this module. Rhonda and I had a wonderful time putting this together. We also hope you enjoyed our little song and dance at the end. That's for your entertainment and for your enjoyment with your students. Also, if you have any curious students who are wondering, "Did they actually figure out what the average note was that Dan sang or the average number of taps that Rhonda tapped?" We actually did! And we've got a file on the website once again that has the full details, so feel free to discuss that over with your students on how we did it. And again, if you have other flaws of averages that come to mind, please feel free to share them or other examples that work out in between the two segments. Please don't hesitate to use those as well! We hope you had a wonderful time with our module, and once again Rhonda and I had a great time putting it together. Take care and have a good day!

END OF TRANSCRIPT