BLOSSOMS MODULE:

MYSTERIES OF MOTION

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There are so many things that move around us. The wind blows, birds fly, trains run on tracks, the cars on the road and you can see that there's motion everywhere. But understanding the nature of motion has taken humanity a very long time. In fact, until 400 years ago people used to think that a body moves, and keeps moving, only if there's something pushing at it. So if you were to stop that thing from pushing, it would come to a rest.

Now Aristotle, who was a very wise man, had a theory of motion which today you and I might find a little peculiar. He said that a rock will fall to the earth because that rock is a part of mother earth and mother earth is pulling it downwards.

"Take an arrow," he said. "An arrow needs a push from the bow string, or a spear needs to be thrown. But then what keeps it moving? Well, as the spear moves forward the air rushes behind and so it's pushing at the spear."

Well, what about the planets in the sky? Again, people used to think there's got to be something pushing at the planets – so it has to be angels or it has to be some kind of supernatural creatures. But that was a long time ago.

Then came the modern age, that of Galileo and Newton. Today we have a very different view of motion, what causes it, what sustains it. This is something that you and I are going to talk about today. But we won't just talk, we shall use the ideas that are developed to actually do a few experiments and know that what we're doing is indeed sensible.

So today I want to introduce some basic concepts. But first of all I want to talk about mass. Mass is really a measure of the quantity of matter. If I have a ball, this ball is matter and this ball has a certain mass. If I had a bigger ball, it would be something with a larger mass. All right, so mass is clear.

Now there's another thing – and that is momentum. Momentum is like a measure of the "quantity of motion", and it's a product of two factors. First of all mass, which I'm going to call M, multiplied by the velocity at which that mass is moving, which I'll call v. Mathematically, momentum is M x v. So if this ball is moving faster, its velocity is greater and the momentum is correspondingly greater. That's if it was moving this way forward. But if it's moving backwards, well then it will be M x (–v). The momentum is now negative. So you could say momentum is a measure of the amount of motion that you have.

Now we go onto another concept – that of kinetic energy. Kinetic energy is (½M x V x V) or ($\frac{1}{2}MV^2$). Kinetic energy – the word "kinetic" comes from the Greek *kinesis,* which means motion. So you could say that this is motional energy. These are two new concepts that we've learned today: momentum and kinetic energy.

Before we break, I want to ask you a question. Suppose there's a motorcycle and a truck and the truck weighs 100 times more than the motorcycle. If they both have the same amount of momentum, how much kinetic energy will the motorcycle have in comparison to the truck? When we come back I'll expect you to have the answer.

You will remember that I made two definitions, one of momentum and the other of kinetic energy. But what use are definitions unless one uses them? The use of the definition of momentum comes really from Newton's First Law, which says that if you have a body that's far away from every other body, then its momentum remains constant. Now the real use of this comes from the fact that it just doesn't have to be one body alone. Suppose you have a whole bunch of bodies. You have 2 bodies or 3 bodies or 20 bodies, but these are all somehow far away from everything else. Well, the total momentum of this collection of bodies is going to be a constant, provided this collection of bodies is far away from every other set of bodies. OK. That is called the conservation of momentum.

The reason I talked about kinetic energy is because kinetic energy is another form of energy. As you know there is heat energy, chemical energy, electrical energy, and so forth and all of these energies are able to convert from one form into the other. Now kinetic energy will be conserved if two bodies collide with each other, provided that it's a special kind of collision.

Now let me come to the subject of collisions, which we're going to talk a lot more about during this lecture. Let's say that I have one cricket ball and another cricket ball. When they collide like this, well this ball doesn't really lose its shape. It remains as it was. And so you have a situation where kinetic energy is more or less conserved. So what you see is that this ball and this ball after they've collided, they have kept moving. And if it is a perfectly elastic collision, then the kinetic energy before the collision and after the collision are exactly the same. That will happen when absolutely nothing happens to the balls.

On the other hand if I have eggs and these eggs collide with each other or with something else, well, it will be a different story. Surely they will not remain in their present form. And kinetic energy will not be conserved.

So let's see. Here's the ball that I'm going to drop. It dropped and OK nothing very much happened to it. Of course the energy was lost in various forms because you heard that sound, so certainly some of the energy was given off in the form of sound. But most of the energy remained in the form of kinetic energy. On the other hand look at this egg. As I drop it it broke, so this was a highly inelastic collision.

Now let's come to this issue. Under what circumstances is a collision elastic and under what circumstances is it inelastic?

Before we come to the break I want to ask you this question. Does the degree of elasticity of the collision depend upon the speed at which bodies collide with each other? Think about that.

A second question is related to the first. Can you think of any situation or any system where you have a perfectly elastic collision?

Again I repeat: an elastic collision is where the initial kinetic energy and the final kinetic energy are exactly the same and an inelastic collision in one in which they're not the same. So kinetic energy is lost. Of course that doesn't mean that the energy has been lost, it has just gone missing. This means that it's turned into another form. So when we return we're going to use the fact that certain collisions can be very elastic and then we'll use that to go further.

Let's apply the two laws that we've learned, namely the law of conservation of momentum and the law of conservation of energy, to a slightly different situation. What I want to consider as our new situation is the mass, M, moving with velocity V. It hits the mass M, which is at rest and then after the collision the first one moves off with velocity V_1 and the second with velocity V_2. Now I don't know what V_1 and V_2 are, but the law of conservation of momentum and of energy are going to tell us what they are. So let's apply them.

First, the initial momentum, that is to say, the momentum before the collision was MV. The momentum after the collision is $MV_1 + MV_2$. So $MV=MV_1 + MV_2$. We can cancel the M and get $V= V_1 + V_2$.

Now let's apply the conservation of energy.

So $\frac{1}{2} MV^2 = \frac{1}{2} MV_1^2 + \frac{1}{2} MV_2^2$.

And again we can cancel the M's, and we get $V^2 = V_1^2 + V_2^2$. We've now got to solve this equation and this equation. How do we do that? Let's square the first one. That way we get V^2 and then equate it to this V^2 .

So $V_1^2 + V_2^2 = V_1^2 + 2V_1V_2 + V_2^2$

Now I can cancel this and this, with this and this, and that gives us $V_1V_2 = 0$. There are three possible solutions of this. V_1 and V_2 can both be 0, but you know that doesn't make much sense because you wouldn't be then able to satisfy this equation $0 + 0$ cannot be V.

Then we can have V_1 as zero and V_2 as non-zero. And let's see what V_2 has to be. Well, it's clear that if V_1 is 0, then V_2 has to be V. The solution over here, $V_1 = 0$, $V_2 = V$ obviously satisfies this.

What this is saying is that the first body comes to a stop i.e. zero speed, and second body moves off with speed V equal to the initial one.

So now let's go out and try and see if this is actually the case. And now the only thing that we have to do is go back over here. And now this ball is going to come and strike the other ball with speed V. If what I've said is right, then this one comes to a rest and this one moves off at the same speed that this hit it with. OK let's go. (balls moving and hitting each other.) That's exactly what we expected and that's what happened. And one more time. (balls moving and hitting each other.)

When we come back after the break you're going to see not 2 but 3, 4, and 5 balls. So ask yourself what will happen in that case? I want you to predict on the basis of what we have learned right now, what will happen when there's a third ball, a fourth ball, and a fifth ball? This first ball comes and strikes this one. Let's see if you can guess the answer by the time we come back.

As we predicted earlier if one ball comes and strikes another ball and if the collision is elastic then the first ball comes to a stop and the other one starts moving off at the same speed as the first ball. This is what we saw over here, i.e. that the solution is $V_1 = 0$ and $V_2 = V$.

All right, now let's see what happens when there's a third ball involved. I'm going to move these two away from the scene. Here will come the first ball, hit the second and that hits the third. (demonstrates). Now that's exactly what we'd expected right? That the first one communicates the energy and momentum to the second, which then communicates to the third. Let's repeat with now four balls and see how that works out. (demonstrates) OK. So you get the idea there. And now let's try for five. Here's five. (demonstrates)

Now there's one last thing before we go onto something else. What if two balls are coming together, one from this side and the other from this side? Let's see what that does. (demonstrates) Of course this could go on forever, it's a very amusing plaything. But basically it's just one ball hitting another and we're just using what we've learned over here again and again.

But now I want to come to something slightly different. What I'm going to do is ask you to complete a calculation that I'm just about to start. And so in here, here is a mass M, which comes in with speed V. But now it's going to hit something that's twice as massive, i.e mass 2M. And I want to find out what happens after the collision. So I'll set up the equations and you solve them. The initial momentum is M into V, plus 2M into 0. That is equal to the final momentum. The final momentum of this one we're going to call V_1 as before. And the final momentum of the second one is MV_2.

Conservation of momentum reads $MV = MV_1 + 2MV_2$.

And conservation of energy reads $\frac{1}{2} MV^2 = \frac{1}{2} MV_1^2 + \frac{1}{2} (2M)V_2^2$.

So these two equations are just $V = V_1 + 2V_2$ and the second one is $V^2 = V_1^2 + 2V_2^2$.

Again, you can solve this as I did before but you're going to do it and I'll give you the answer. The answer is that $V_2 = 2/3V$ and $V_1 = -1/3\ V$. So check this out.

Just look at what this is saying. Its that the first ball, i.e. the one that strikes the big one, is going to recoil at a speed which is minus 1/3 V. The final speed of the heavy ball is going to be 2/3 V. Actually you can check very easily that this satisfies this. Just put this in here, so that's $-1/3 + 4/3=1$. That satisfies the equation. There is recoil in the backward direction. The struck ball is going off with a speed less than the speed with which it was hit. So the heavier body has absorbed some of the momentum and energy of the lighter one, although it hasn't absorbed it all.

Based on this example I'm going to ask you a question. I have over here a little ball that I'm going to drop to the floor and you see that it drops and comes back. When it hit the floor it was moving downward and when it rebounded it was moving upward, so obviously there's been a change of momentum. Change of momentum! Well, does that mean momentum is not conserved? That's my question.

Similarly, if I throw this against a wall, again I see a change in momentum. It was moving this way, then it went that way. So what's going on over here? Think about this during the break.

I hope you had time to reflect on the question that I had asked you before the break because it's a very important question. To remind you, it was this situation where I drop a ball and the ball strikes the ground, comes up. Apparently there is no conservation of momentum, but why? Well, the reason is that yes, this ball's momentum has changed, but so has the momentum of the earth. When this ball struck the earth, the earth was forced down. It absorbed the momentum. It gained momentum as a consequence of the ball striking it. That sounds a little strange, doesn't it? I mean, the earth is so big, how could a tiny little ball like this change the momentum of the earth? But that is exactly it. The total momentum, which means the sum of the momenta of the ball plus the earth doesn't change.

OK. Still it is a fact that the momentum of this ball changed. And Newton's law says that the rate of change of momentum $\Delta P/\Delta T$ is the force that acts on a body. Let me say this again: if the momentum of a body changes by amount ΔP in time ΔT that is because a force F has acted upon it.

Now let me use this to explain something that is of very common knowledge and I'm sure will be of interest to you if you're a cricket player. Before that let me write this equation as $\Delta P = F \times \Delta T$. Again what this means is that if a force F acts for a time ΔT, a change of momentum ΔP occurs. Note one thing: F can be large and ΔT can be small to give ΔP. Or F can be small and ΔT can be large. This can give the same ΔP.

So now let's come to the question of the cricket ball. As any good fielder knows if this is a ball that's coming toward you and you have to catch it in this hand, well you're going to let the hand go back a little bit. In this way ΔT – which is the time for the ball to come to a stop – is allowed to be longer so that the force is smaller. The final momentum is, of course, zero because the ball has to come to a stop in your hand. The point is that if you move your hand backward, you allow a larger time and hence a smaller force. Obviously a smaller force hurts your hand less.

Let's take another example of this. Now you remember that I dropped an egg on this table and the egg broke. I'm not going to do that again! But let's take a softer surface. OK. I'm going to drop it from pretty much the same height and the egg didn't break. Why not? Well exactly the same reason. The reason was that a smaller force acted for a larger time and that gave the same change in momentum, that is the initial momentum minus the final momentum, final momentum being zero because it came to a stop. Hence, what we see is that this egg did not break. We had well nearly an elastic collision.

There is a lot more that we can talk about but we've run out of time. All I'm going to say is that Newton's laws are so fundamental to physics and so fundamental to having created the world around us that practically nothing in this world would be in the shape that it is if it wasn't for our having understood the mystery of motion.

Newton's laws require critically the definition of momentum. So let's go back to that just very briefly. You remember that I had defined momentum P as the mass of the body into the velocity. And then of course rate of change of momentum is the force. Is it true that this holds all the time, everywhere, and in any situation? And my answer to that is it's 99.99% true, but it's not

100% true. I say 99.9999…. because we just need look at all the things that we see around us. We see cars or rockets or planes or whatever. They're not moving anywhere close to the speed of light. But what Einstein showed—and this was in 1905, more than 100 years ago—was that if some bodies start moving close to the speed of light, for example electrons, this formula over here doesn't work and has to be modified. But it's only a tiny modification. All you do is multiply this thing by a certain factor. It's called the gamma factor.

But I'm not going to talk about the gamma factor. Think of this as one for all practical purposes here on earth. In the sub-atomic world of atoms and electrons, this gamma can be very large. This difference leads to nuclear energy and to understanding how the stars shine, where they get their energy and light from. But that's another story. That's all for today. I hope that you understood some of the basic concepts that we introduced. Of course there's a whole lot more!

TEACHERS' NOTES

These remarks are only meant for teachers. I hope you enjoyed this program. There are lots of other things that you can do with your students in teaching mechanics. Of course all the material that you saw you can find in any decent textbook in physics. The sort of extensions that you can look at are for example, applying the conservation of momentum to two dimensions and then to three dimensions. This is not difficult because whether in two dimensions or three dimensions, basically you've got to conserve momentum in each one of those dimensions.

Let me come to the two questions, which I think are a little thought-provoking. The first is: does it matter how hard you collide two bodies in terms of the collision being elastic or inelastic? The answer is yes. Suppose I take two steel balls and they collide at a reasonable velocity. Well, they bounce back with almost that same velocity so it's an almost elastic collision. But now imagine that I was to throw those steel balls at each other with some huge velocity. If it was absolutely huge the balls would just disintegrate, even the iron or steel would be shattered. So obviously there is a limit upon the speed at which you can throw these two balls together and expect them to remain the same. So collisions between large objects are elastic, but always up to a point.

Which then brings us to the next question: is there such a thing as a perfectly elastic collision anywhere in the world? The answer is yes. Something that's absolutely impossible to change is a fundamental particle – like an electron or perhaps a quark. But let's talk about electrons. Take two electrons, strike them together as hard as you like. Well, they are going to remain electrons. Nothing is going to happen. And so that's an example of an absolutely, perfectly elastic collision.

Finally let me discuss a gas. The gas has got lots of atoms and molecules and they're all colliding with each other. One goes and strikes the other, and there is no loss of energy because those are elastic collisions. So that gas is not losing its energy, it's just that the energy of one atom or one molecule gets transferred to the other and then another and so forth.

The subject of collisions is really very important to physics, not just in terms of let's say cars colliding with each other, although that's a good example to use in your classes. But also in terms of what goes on inside gasses, inside solids, and inside all of matter. That's why when we begin to study physics, the first thing is mechanics. Mechanics is absolutely fundamental and I hope you have a good time teaching that to your students.

END OF LESSON