## $\underline{\text { Blossoms - How Big is a Mole v4 }}$

[MUSIC PLAYING]
STUDENT: But it's not fair. She asked me how many carbon atoms there are in 12 grams. And I gave her the right answer-- 6.02.

ALAN CROSBY: Really? Those are some pretty massive carbon atoms you have there at 2 grams apiece. Not to mention you magically invented the 0.02 atom.

STUDENT: OK. Whatever. 6.02 times 10 to the 23rd. I gave to the right number though. Isn't that what matters?

ALAN CROSBY: There's a big difference between 6 atoms and 6 times 10 to the 23rd atoms. But it's easy to lose track when you're talking about so many very tiny particles. But you're not alone, Maya. Many students and adults have a difficult time imagining Avogadro's number of tiny particles.

Perhaps it would help if we imagined a more sizable particle, such as a ping pong ball. Atoms and molecules are too small to see or even imagine. But a ping pong ball, that's a particle you can really wrap your head around. And look what just one power of 10 does. Here are 6 ping pong balls, or 6 times 10 to the first, or 60 ping pong balls.

STUDENT: Wow.

ALAN CROSBY: Now imagine 6 times 10 squared, or 600, or 6 times 10 cubed, 6,000. 60,000 . Is that a lot of ping pong balls?

STUDENT: Definitely. That's a lot.
ALAN CROSBY: What would you consider a lot of ping pong balls?
STUDENT: Hm. Maybe the amount of ping pong balls to fill this entire classroom. That would be a lot of ping pong balls. I don't know how many, but it must be tons.

ALAN CROSBY: What a great idea, Maya. But let's get some help. So your task is to estimate the number of ping pong balls required to fill your classroom.

Oh no. We don't need one of those. We want to estimate an order of magnitude, a power of 10 .

STUDENT: Oh, you mean a ballpark number?
ALAN CROSBY: That's right. An estimate. So your task is to estimate the number of ping pong balls required to fill your classroom. Now your classroom may not be the same
size as our classroom. But I'll bet if we work on an order of magnitude that we'll come up with the same number.

Welcome back. I'm Alan Crosby, a teacher here at Newton South High School in Newton, Massachusetts.

STUDENT: Hi, I'm Maia Fefer, a chemistry student at Newton South High School.
ALAN CROSBY: Now Maia has her estimates ready. Do you have your estimates ready? Let's see just how many ping pong balls it takes to fill a classroom.

STUDENT: Assuming that the average ping pong ball has a diameter of 40 millimeters, I estimate it will take about 1 times 10 to the sixth, or 1 million ping pong balls to fill the room. Wow. That's a lot of ping pong balls. If I had a million dollars, I'd be rich.

ALAN CROSBY: You're right, Maia. That sounds like a lot of ping pong balls. But is it really? We started thinking about ping pong balls because we wanted a model about Avogadro's number. Are we close to Avogadro's number?

STUDENT: No.
ALAN CROSBY: No, not really. Maia, we've got to think bigger. Our classroom only holds a million ping pong balls. We need a bigger model.

STUDENT: OK. What if we fill up all the classrooms in the entire school? That's got to get us near to Avogadro's number of ping pong balls.

ALAN CROSBY: Not even close. Maia, you've got to think bigger.
STUDENT: OK. What if we fill up all the classrooms in all the schools in the city of Newton. That's got to get us to Avogadro's number.

ALAN CROSBY: Not even close. You've got to think bigger.
STUDENT: OK. How about we fill up all the classrooms in all the schools in all the towns and cities in the state of Massachusetts?

ALAN CROSBY: Perhaps. But I think we need to think even bigger.
STUDENT: OK. How about we fill up all the classrooms in all the schools in all the cities and towns in all the states in the entire country?

ALAN CROSBY: All right. Now you're talking. Could you help us out? How many ping pong balls would it take to fill all the classrooms in all the schools in all the cities and towns in your entire country?

STUDENT: Wait a minute, Mr. Crosby. It's going to take these students forever to count up the individual ping pong balls.

ALAN CROSBY: Well, not necessarily. We estimated it would take a million ping pong balls to fill up a single classroom. So each time we fill a classroom, that's a million or 10 to the sixth ping pong balls.

STUDENT: Oh. I see. So if there are 20 classrooms in Newton South High School, then there are 20 classrooms of ping pong balls, so there are 20 million ping pong balls.

ALAN CROSBY: Exactly. You can use a classroom as a counting unit. So if we assume that there are 20 classrooms in the school, and 20 schools in the town, and about 100 Newton-sized towns in the Commonwealth of Massachusetts, we're already up to 40,000 classrooms. So you see, it shouldn't take our students too long to estimate how many classrooms there are in their country.

STUDENT: Great. Maybe you can help us with our estimates. Your next activity is to estimate the total number of ping pong balls needed to fill up all the classrooms in all of the schools in all of the cities and towns in your entire country. I hope you can tell me if we're getting close to Avogadro's number.

ALAN CROSBY: All right. Our students are back with their estimates of how many ping pong balls it will take to fill all of the classrooms in all of the schools in all of their cities and towns in their entire country. Now let's see how their estimates compare to our estimates.

STUDENT: OK. Let me see if I've done this correctly. We already estimated that it takes 1 million ping pong balls to fill up 1 classroom and that there are approximately 40,000 classrooms in the state of Massachusetts. Since there are 50 states in the United States, that puts our estimate at 2 million classrooms full of ping pong balls, or 2 times 10 to the 12 th ping pong balls.

ALAN CROSBY: That sounds right to me. But the United States is a pretty big country. Carrying out this estimate on any country in the world, though, should get you a number somewhere between 10 to the 11th and 10 to the 13 th ping pong balls. Does that sound like a lot of ping pong balls?

STUDENT: Well, yes. It does sound like a big number. Yet it's nowhere near Avogadro's number. I'm beginning to have trouble imagining so many ping pong balls at once.

ALAN CROSBY: It gets cumbersome to keep track of such large quantities of things. That's why we use scientific notation to represent very large numbers. The trade off is you have to remember to write down both parts of the number.

Imagine if someone had said to you, it only takes 2 ping pong balls to fill up all the classrooms in all the schools in the entire country.

STUDENT: That would be pretty far off. I mean, it takes 2 times 10 to the 12 th power to fill those classrooms. Oh. I guess I'm beginning to realize why my teacher took off so many points when I left off the times 10 to the 23 rd when I was speaking about Avogadro's number of carbon atoms.

ALAN CROSBY: You know, Maia, leaving off the 23 zeroes in Avogadro's number is an example of something I see more and more these days with many students. Many seem to have forgotten how to obtain those back-of-the-envelope approximations of quantities, relying instead on their calculators. But without an intuition for order of magnitude, the decimal point can easily be misplaced and not noticed.

For instance, just one skip of a decimal point in giving medication to someone could result in a severe medical distress instead of healing medicine. It's always helpful to have an idea or a sense about what you are expecting as a result. With that in mind, let's see if we can get a sense for Avogadro's number.

Even though we tried to think big, we still weren't able to get to Avogadro's number of ping pong balls, even by filling up all the classrooms in all of the schools in our country. So let's try looking at the problem a different way. Imagine that we start with Avogadro's number of ping pong balls. Let them spill all over the Earth.

Would they cover the Earth? One layer deep? Multiple layers deep? Or would they cover Mount Everest?

STUDENT: Imagine that. A sea full of ping pong balls. But that would be a tough estimate. Where would we start?

ALAN CROSBY: Well, let's see if the students can help us out. Can you estimate how deep the layer of ping pong balls would be if Avogadro's number of ping pong balls was covering the surface of the Earth? For this estimate, it might help to imagine the ping pong balls as being cubes so you can calculate the volume of a single layer.

It would also be helpful to know that the Earth has an average radius of about 6,400 kilometers and the surface area of the Earth is roughly 5 times 10 to the 14th meters squared. Remember, we're still using estimates, orders of magnitude. Let's see if we can reach the summit of Mount Everest.

So let's see how those estimates turned out. Maia, we need to see how our estimates compare to those of our students estimates.

STUDENT: Wow, Mr. Cosby. According to my estimates, a mole of ping pong balls would cover the Earth in a layer 80 kilometers deep. That's really, really deep.

ALAN CROSBY: In fact, that's almost 10 times the depth of Mount Everest. It's literally out of this world. It's beyond the stratosphere and into the mesosphere.

So a mole of ping pong balls is a very, very large volume indeed.
STUDENT: But in my class, we also learned that 18 milliliters of water contains a mole. And it all fits into this little, graduated cylinder

ALAN CROSBY: And you're right. The 18 milliliters of water in this graduated cylinder contain Avogadro's number of water molecules, just like a beach ball you might use on the beach contains Avogadro's number of gas particles. Imagine how many moles of gas there were to fill a lighter than air ship like the Goodyear Blimp.

STUDENT: Oh, I get it. Avogadro's number of water molecules doesn't cover the Earth because molecules are very, very small compared to larger particles such as the ping pong ball.

ALAN CROSBY: Exactly. If we were to try to count Avogadro's number of particles individually, it would take longer than the universe has been around. That's why we lump them together into groups called a mole. A mole contains 6.022 times 10 to the 23rd particles.

A mole isn't very useful for talking about macroscopic objects. For as we've seen, even a mole of a small object, like a ping pong ball, would cover the Earth many, many times over. But in science, we use moles to measure quantities of atoms and molecules because atoms and molecules are very, very small. Without using the concept of a mole, science would be very cumbersome indeed.

STUDENT: Definitely. Who wants to go writing Avogadro's number with all 23 zeroes at the end? My arm gets tired just thinking about that.

ALAN CROSBY: You know, Maia, we spent a lot of time estimating today. But do you think our students would be up for one more challenge?

STUDENT: I think so. What did you have in mind?
ALAN CROSBY: Well, we talked about how small water molecules are, but we never really quantified that. So I'd like us to take a look at something very small. Could you read this graduated cylinder for me?

STUDENT: Sure. It looks like 18 milliliters.
ALAN CROSBY: 18 milliliters. That's right. So your job is to use 18 milliliters and Avogadro's number, 6 times 10 to the 23rd power, to estimate the size of an individual water molecule.

All right, Maia. Let's see how our estimates compared to the estimates of our students.

STUDENT: It looks like the average water molecule would have a radius of about 2 times 10 to the negative 10 meters. That's so small. I have to say, Mr. Crosby, I'm really glad that I came to talk to you about my test. I really learned a lot today. I don't think I'll ever forget to write the times 10 to the 23 rd at the end of an Avogadro's number.

ALAN CROSBY: I'm glad to hear that, Maia. We've done a lot today. We've learned about Avogadro's number. It's a very large number of particles. And the reason it's so large is because atoms and molecules are so very, very small.

We spent a lot of time also understanding the significance of the order of magnitude, from very large numbers like Avogadro's number, almost 10 to the 24th power, down to very, very small numbers like the dimension of an atom or molecule, on the order of 10 to the minus 10 or 10 to the minus 9 meters.

Thank you for joining us today. Maia and I have had a great time estimating the number of ping pong balls it would take to fill our classroom and imagining a mole of ping pong balls surrounding the Earth. We hope you've had a good time, too, and that you've learned a lot.

ALAN CROSBY: Hi. I'm Alan Crosby.
DR. JESSICA SILVERMAN: I'm Dr. Jessica Silverman.

ALAN CROSBY: We are science teachers in the science department at Newton South High School in Newton, Massachusetts. And we're happy that you joined us on our lesson, How Big Is a Mole?

DR. JESSICA SILVERMAN: So we created this lesson because we wanted to help give students an appreciation for the magnitude of a mole. Because as we saw today, it's really, really large. And it can be very hard for students to get their head around what 10 to the 23 rd really means. We also wanted to give students some practice with estimations and getting a number sense. Because it's one thing to punch numbers into your calculator, but it's a really great skill to be able to just estimate things on the fly.

ALAN CROSBY: One of the objectives that isn't written and wasn't really discussed is that you hope that students begin to learn how to estimate their answers such that when they're doing a test, they have the ability to question what their calculator printed out. Did I punch this in right? Because that answer just can't be true. So hopefully some of what we're doing in terms of estimations and evaluations of the order of magnitude will bleed over into other aspects of their learning.

DR. JESSICA SILVERMAN: So we think that this is a really nice, accessible lesson because it doesn't require a lot of prerequisite information. So it's great for students who are currently taking chemistry now or even as a nice review for students who have already taken a year of chemistry. But we feel like it doesn't really require an extensive
math background. This is really accessible to students of all different types of math abilities, especially if the assignment is done in groups, the way that we envision it.

ALAN CROSBY: The activities are designed to be worked in groups of two or three students. We found, actually, that three students is the best number. Let the students work. Just monitor to make sure that they're not pulling out their calculators and they're not trying to "cheat," if you will. It's best if they really try to estimate.

And we purposely have not given them a lot of information to start with. We've held up ping pong balls in our hands for them to estimate the size and have them estimate the volume of their room. And at the end of the first activity, which will take the longest, the first activity is the longest, the groups of students, if you can collect all of their results onto a board and let them all view them. So as the students write their numbers on the board and they start viewing their own numbers and the numbers of there peers, they'll see that in many cases when you round those to one significant figure, that they have exactly the same number.

Although none of the activities are about significant figures. Significant figures do come into play. The estimates that are used here are only good to one significant figure. So if and when students start writing out multiple significant figures as they are prone to do, you can open up a discussion as to why only one significant figure is relevant.

DR. JESSICA SILVERMAN: So we hope that your students really enjoy this activity and learned a lot. And if so, if they would like to do a further activity, we have a great poster project that we assign here to our students at Newton South High School. So this is titled the How Big Is a Mole poster. And we try to get students to think about what 1 mole of a familiar object, like a coin or a dog-- we try to get them to compare that to either the area, the mass, the distance, the volume, of something very, very large.

So for example, if I had a mole of pennies and I stacked them on top of each other, how many times would I go to the sun and back? So something like that. So that's been a pretty good project for your students.

ALAN CROSBY: It has been a great project. And they've learned a lot. The creation of the posters is one aspect of it. Actually doing the calculations and doing the research to compare is another aspect of it. But the overarching hope we have is to help students really, really understand the magnitude of Avogadro's number. Because it truly is an inconceivably large number.

DR. JESSICA SILVERMAN: Absolutely. And I would just add that I also did this project with my students as a shorter exercise. If they don't want to make a poster, you can just give them a list of some suggested calculations. So you could use the coin example. And just say to them, how many times would you go to the sun and back? Or if I had a mole of dollar bills, how many times would it wrap around the Earth? Something like that. So even just some quick calculations like that, my students started to say, wow, this really is huge. It's amazing. So there's a lot of different ways that it can hit home for
students. So we both want to thank you very much for choosing this lesson and taking the time to do this with your students.

ALAN CROSBY: We would also appreciate any comments that you might have.
Comments can be left for us on the BLOSSOMS website.
[MUSIC PLAYING]

