## Successive Approximations as a tool to Measure Distances

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## Video 1: Definition of the Problem

, We see ants walking on the curved side of the Puente Atirantado.


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## Video 1: Definition of the Problem

, We notice that the ants that are walking are getting smaller and smaller.


## Video 1: Definition of the Problem

, We want to know what you would do to find:
, How much does the curved side measure?
, How many ants would cover the curved side if they become increasingly smaller by 0.001 each time?


Video 2. Sheets of Paper Activity. Successive Approximations.

## Succesive Approximations



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## Succesive Approximations



A


B


C


D

## The video will be the answer (There needs to be an animation of this)

, Select two squares and say they are the sheets

) Leave one as is, and add halves made from the other square:

$$
\text { ) }+\quad+\quad+\quad+\ldots=2 \text { sheets }
$$


$\square$
$\square$

## The video will be the answer (There needs to be an animation of this)

, One of us continues to explain in the video::
) that

$$
1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\frac{1}{128}+\frac{1}{256}+\ldots .=2
$$

, These fractions are known as successive approximations.
, How could you use this to solve the problem?
, Have them respond orally. Tell them we are going to work on a smaller problem.

Video 3. Rope Activity: Formula for Distance.

## Video 1: Definition of the Problem

, We notice that the ants that are walking are getting smaller and smaller.


How much does the longest rope measure?


How much does the rope measure, from cross to cross?

¿How much does the rope measure, from cross to cross?
The scale is $1 \mathrm{~cm}=0.5 \mathrm{~m}$. Use successive approximations with the formula for distance.


Formula for Distance (Video of its construction)
(2) Formula for Distance (Video of its construction)

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$\pi$ Formula for Distance (Video of its construction)
 Formula for Distance (Video of its construction)

$\pi$ Formula for Distance (Video of its construction)

$\pi$ Formula for Distance (Video of its construction)





Formula for Distance (Video of its constructic 7 )



Formula for Distance (Video of its constructic 7 )



Formula for Distance (Video of its constructic 7 )








## Formula for Distance (Video of its construction) Let's apply the Pythagorean Theorem



## Formula for Distance (Video of its construction) Let's apply the Pythagorean Theorem




## Formula for Distance (Video of its construction) Let's apply the Pythagorean Theorem

How much does the rope measure, from cross to cross? The scale is $1 \mathrm{~cm}=0.5 \mathrm{~m}$. Use successive approximations with the formula for distance.

Video - Doing successive approximations of the rope, using the distance.


Video 4. The Scale

## Summarizing the video

, We know what successive approximations are
, We know the formula for distance
, We know how to use successive approximations to measure a curved surface
, Now we only need to find the scale

Héctor in the Puente Atirantado We know that Héctor is ??? tall. (Héctor, what is your height?)


## Problem

, We want to know what you would do to find:
, How much does the curved side measure?
, How many ants would cover the curved side if they become increasingly smaller by 0.001 each time?


Video 5. The Solution

## The solution includes:

, Turn the Puente Atirantado around
, Apply Successive Approximations with the distance formula (Excel, Mathematica)
, Apply the scale
, Calculate the number of ants (Excel)

## Puente Atirantado



## Puente Atirantado




## Puente Atirantado



Puente Atirantado


## Puente Atirantado

| No. | X | Y | PMX | PMY | Distancia |
| :---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 1195 | 1930 |  |  |  |
| 2 | 750 | 0 | 707 | 965 | 1980.638 |

$(707,965)$


## Puente Atirantado

## $\pi$

| No. | X | Y | Distancia | Dtotal |
| :---: | :---: | ---: | :---: | :---: |
| 1 | 1195 | 1930 |  |  |
| 2 | 707 | 965 | 1081.374 |  |
| 3 | 750 | 0 | 1980.638 | 3062.011 |

Dtotal $=d 1+d 2$
The formula in EXCEL is
=SUM(D2:d5)


## Puente Atirantado

| No. | X | Y | 0 |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1195 | 1930 | 120 |  |
| 2 | 940 | 1448 | 1200 |  |
| 3 | 700 | 965 | 100 |  |
| 4 | 630 | 483 | ${ }_{\text {son }}^{20}$ | 碞 |
| 5 | 750 | 0 | - | 5 |

## Puente Atirantado

## $\pi$

| No. | $x$ | y | Distancia | Dtotal |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1195 | 1930 |  |  |
| 2 | 940 | 148 | 54.7392 |  |
| 3 | 700 | 965 | 538.8935 |  |
| 4 | 630 | 483 | 487.513 |  |
| 5 | 750 |  | 497.1984 | 2069,382 |

Puente Atirantado

| No. | X | Y |
| :---: | ---: | ---: |
| 1 | 1195 | 1930 |
| 2 | 1068 | 1689 |
| 3 | 940 | 1448 |
| 4 | 815 | 1206 |
| 5 | 700 | 965 |
| 6 | 645 | 724 |
| 7 | 630 | 483 |
| 8 | 670 | 241 |
| 9 | 750 | 0 |

## Puente Atirantado

$\pi$

| No. | X | Y | Distancia | Dtotal |
| :---: | ---: | ---: | ---: | ---: |
| 1 | 1195 | 1930 |  |  |
| 2 | 1068 | 1689 | 272.8696 |  |
| 3 | 940 | 1448 | 272.8696 |  |
| 4 | 815 | 1206 | 271.7104 |  |
| 5 | 700 | 965 | 267.2575 |  |
| 6 | 645 | 724 | 247.44 |  |
| 7 | 630 | 483 | 241.7159 |  |
| 8 | 670 | 241 | 244.7902 |  |
| 9 | 750 | 0 | 253.9311 | 2072.584 |

Vememino
$\pi$

| No. | $X$ | $Y$ |
| :---: | ---: | ---: |
| 1 | 1195 | 1930 |
| 2 | 1131 | 1809 |
| 3 | 1068 | 1689 |
| 4 | 1004 | 1568 |
| 5 | 940 | 1448 |
| 6 | 882 | 1327 |
| 7 | 815 | 1206 |
| 8 | 750 | 1086 |
| 9 | 700 | 965 |
| 10 | 660 | 844 |
| 11 | 645 | 724 |
| 12 | 628 | 603 |
| 13 | 630 | 483 |
| 14 | 650 | 362 |
| 15 | 670 | 241 |
| 16 | 710 | 121 |
| 17 | 750 | 0 |

Puente Atirantado



## Puente Atirantado



Note for Roberto: See steps 0-6 to see the algorithm that generates the succesive approximations in the Excel document. The next chart is the last graph generated.

The Excel calculation for the measurement of the curved side of the Puente Atirantado is 2075 Units.


## Now let's apply the scale.


$\pi \quad$ Scale: Héctor is 1.70 meters tall


Scale: Héctor is 1.70 meters tall


When we move Professor Héctor horizontally, the points change and we can apply the Pythagorean Theorem.
$\pi \quad$ Scale: Professor Héctor is 1.70 meters tall

$c_{1}=\sqrt{(740-740)^{2}+(30-10)^{2}}=\sqrt{400}=20$
$h=\sqrt{(750-740)^{2}+(10-30)^{2}}=\sqrt{100+400}=22.36$
La escala de $E X C E L 20=1.70 \mathrm{~m}$
$c_{2}=\sqrt{h^{2}-c_{1}^{2}}=\sqrt{22.36^{2}-20^{2}}=\sqrt{99.96}=9.99$

$$
2075 / 20=103.75 * 1.70=176 \mathrm{~m}
$$

According to Excel calculations, the curved side of the Puente Atirantado measures 176 meters.


According to Excel calculations, the curved side of the Puente Atirantado measures 176 meters and 17600 ants are needed.

```
2000 (1)
```


## Fif UNIVERSIDAD TECMILENIO.

## Teacher Guide Segment

## Puente Atirantado


$\pi$
Approximate Length of Arch (Excel)


200

| 311 | $\begin{aligned} & =\text { SQRT(POWER } \\ & ((\text { A3-A2), } \\ & \text { 2)+POWER((B3- } \\ & \text { B2),2)) } \end{aligned}$ | $\begin{gathered} L=-S U \\ M \\ \text { (C2: } \\ \text { C2: } \end{gathered}$ |
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# Succesive Approximations as a Tool to Measure Distances 

## END

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