# LEARNING OBJECTIVES:

1) CALCULATE THE FINAL VEL. OF ROCKET.

2) CALCULATE PERIODS, ENERGIES, ANGULAR MOMENTA OF PLANETS.

3) EXPLAIN IN YOUR OWN WORDS HOW TO TRANSFER LINEAR MOMENTUM FROM ONE MARBLE TO ANOTHER.

# LEARNING OBJECTIVES: (CONT'D)

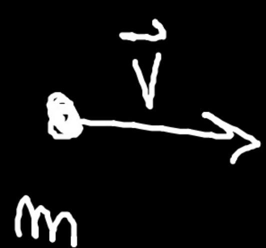
4) EXPLAIN IN YOUR OWN WORDS WHAT GRAVITY ASSIST IS.

5) WORK OUT SPECIFIC EXAMPLES OF GRAVITY ASSISTS.

6) EXPLAIN IN YOUR OWN WORDS HOW CAN A SPACECRAFT LAUNCHED FROM EARTH REACH SATURN FASTER THAN ON FUEL ALONE.

# LINEAR MOMENTUM

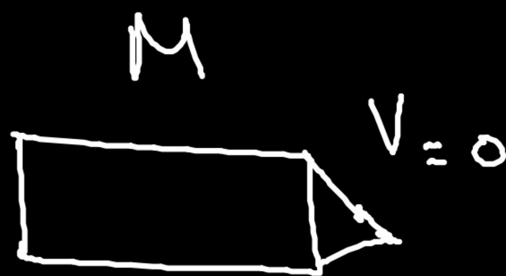
$$\vec{P} = m\vec{v}$$



LINEAR MOMENTUM CONSERVATION:

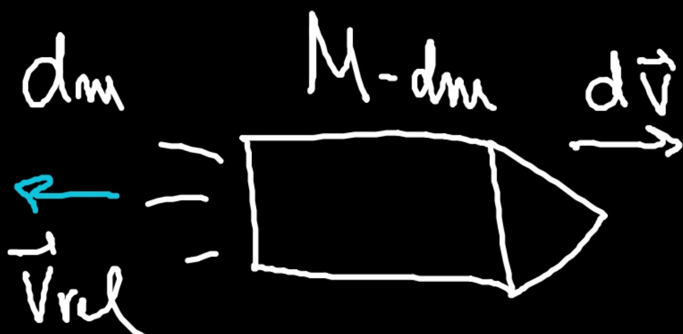
$$\vec{P}_i = \vec{P}_f$$

ROCKET @ rest




$$\vec{P}_i = 0$$

MOVING ROCKET




$$\vec{P}_f = 0$$

$$\boxed{M\,d\vec{v} + dm\,\vec{v}_{rel} = 0}$$

# ROCKET EQUATIONS:

$$\vec{V}_f - \vec{V}_i = -\vec{V}_{rel} \ln \frac{M_i}{M_f}$$

$$T = \left(\frac{dM}{dt}\right) V_{rel} \quad : \text{THRUST}$$

WHERE $\dfrac{dM}{dt} = -\dfrac{dm}{dt}$

EXAMPLE:

$v_{rel} = 3\ km/s, \quad \dfrac{dM}{dt} = 200\ kg/s$

$\therefore$ THRUST IS:

$$T = \frac{dM}{dt} v_{rel} = (3\ km/s)(200\ kg/s) = 600{,}000\,N$$

$$M_{max} = T/g = \frac{600{,}000\,N}{9.81\,m/s^2} \approx 61{,}000\,kg \sim 61\ tons$$

EXAMPLE: (cont'd)

$M_i = 3000\,kg$, of which $2000\,kg$ in fuel

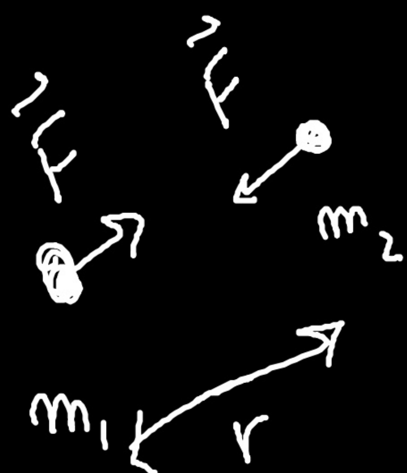
$\frac{dM}{dt} = 0.003\,kg/s$. $V_i = 0$, $V_{rel} = 3\,km/s$

$$V_f - \cancel{V_i}^{\,0} = -(-3\,km/s)\ln\frac{3000\,kg}{1000\,kg} = 3.3\,km/s$$

$$t_{burnout} = \frac{2000\,kg}{0.003\,kg/s} = 185\text{ hours}$$

# NEWTON'S LAW OF GRAVITATION

$$F = G\frac{m_1 m_2}{r^2}$$



NEWTONS $2^{nd}$ LAW:

$$\vec{F} = m\vec{a}$$

ORBITS: NEWTON'S GRAV = NEWTON'S $2^{nd}$

$$\frac{Gm_1m_2}{r^2} = m_1a_1$$

$$\frac{Gm_1m_2}{r^2} = m_2a_2$$

KEPLER'S 3 LAWS:

1) ALL PLANETS ORBIT SUN IN ELLIPSES WITH SUN BEING AT ONE FOCUS.

2) VARIABLE SPEEDS, FASTEST @ PERIHELION
CLOSEST
SLOWEST @ APHELION
FARTHEST

3) $P^2 = a^3$

period

semi-major axis
(distance from center of ellipse to peri or ap-helion)

TOTAL ENERGY (CIRC. ORBIT)

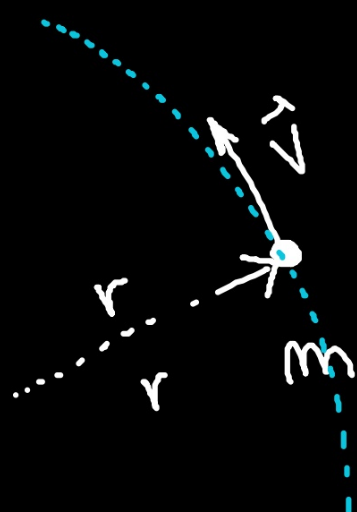
$$E = \frac{1}{2} m v^2 + \left( -\frac{GmM}{r^2} \right)$$

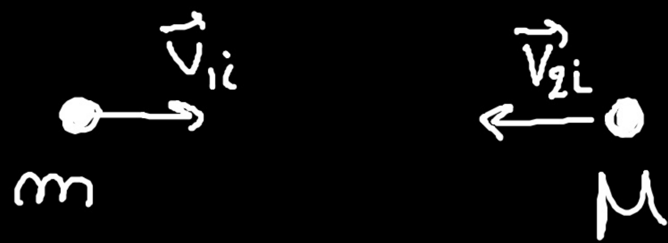
KINETIC ENERGY

POTENTIAL ENERY

# ANGULAR MOMENTUM

$$L = m\,v\,r$$





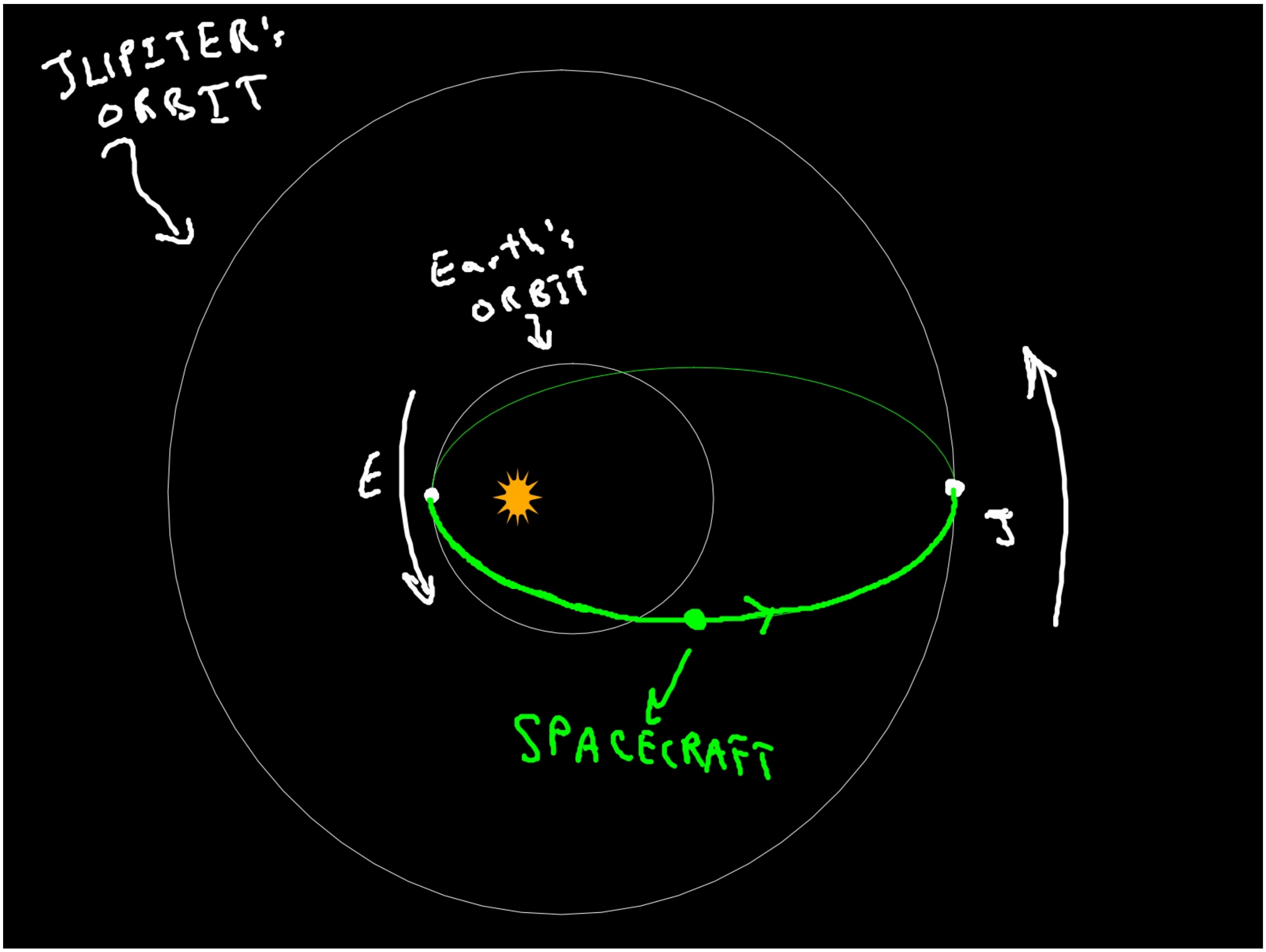

LINEAR MOM. CONS.: $m\vec{v}_{1i} + M\vec{v}_{2i} = m\vec{v}_{1f} + M\vec{v}_{2f}$

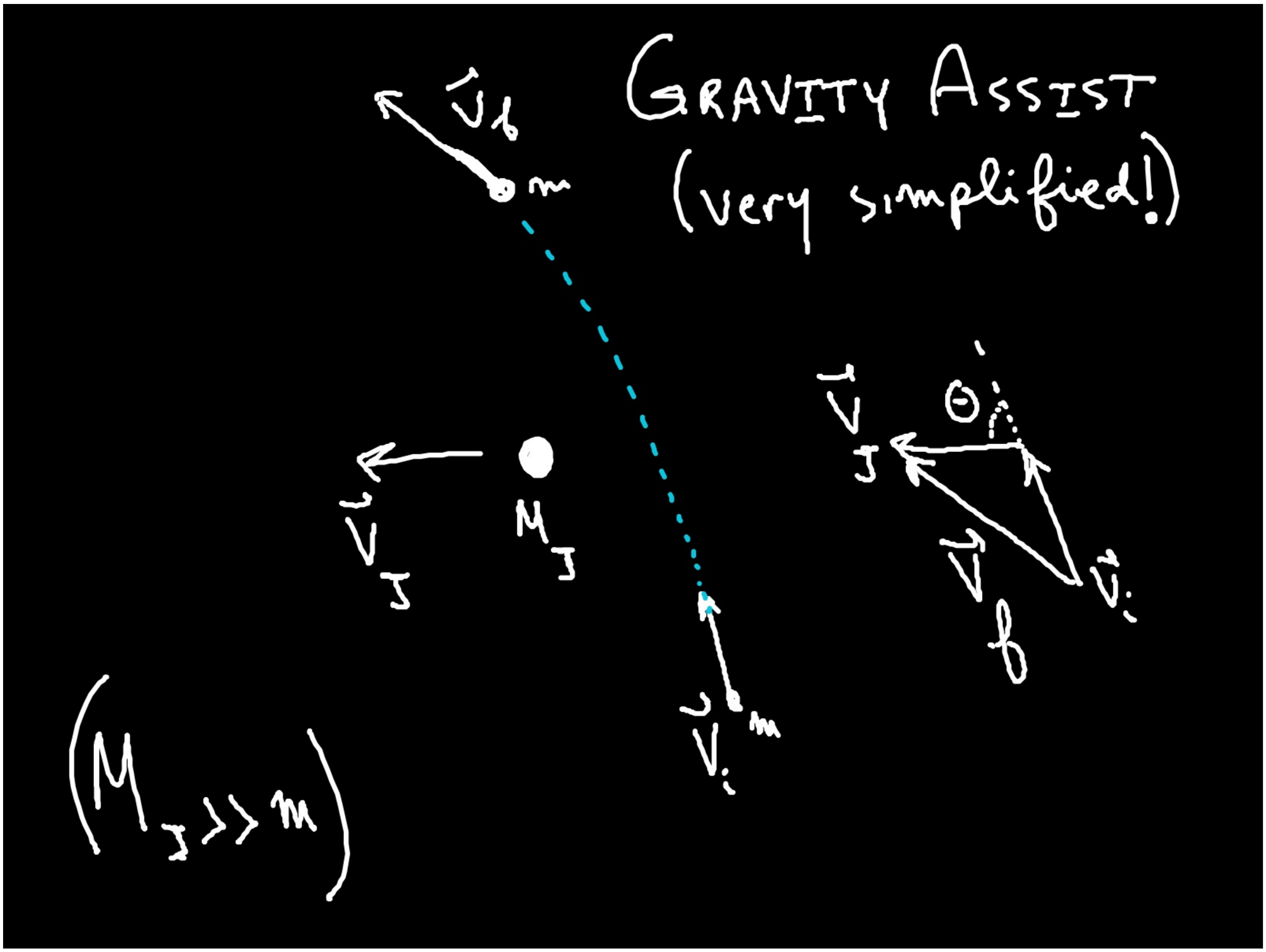
KINETIC ENERGY CONS.: $\frac{1}{2}mv_{1i}^2 + \frac{1}{2}Mv_{2i}^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}Mv_{2f}^2$

$$V_{1f} = \frac{(1 - m/M) V_{1i} + 2 v_{2i}}{1 - m/M}$$

if $M >> m \Rightarrow$

$$V_{1f} = V_{1i} + 2V_{2i} \qquad P_{1f} = m\left(V_{1i} + 2V_{2i}\right)$$


JUPITER's ORBIT
Earth's ORBIT
E
J
SPACECRAFT


GRAVITY ASSIST
(very simplified!)
$\vec{v}_f$
$m$
$\vec{V}_J$
$M_J$
$\vec{v}_i$
$m$
$\theta$
$\vec{V}_J$
$\vec{v}_f$
$\vec{v}_i$
$(M_J \gg m)$