The Parallax Activity: Measuring the Distances to Nearby Stars

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Part I: Preparation

A – Objectives: At the end of this lesson, students will be able to:

1. Measure the parallax angle between a simulated planet and a distant star using a homemade quadrant.
2. Calculate the distance to the star objects using the tangent function.
3. Explain the concepts of parallax angle, astronomical unit, parsec, and astrometry.
4. Apply their knowledge by determining the height of a distant object.

B – Materials list (for one group of four students):

1 meter long wooden ruler, wooden dowel, or thin PVC pipe
Protractor
50 cm of string
Transparent tape
One-hole rubber stopper or other small weight which can be tied to the string
Calculator able to do trigonometry functions and scientific notation
Astronomy planetarium software, such as Stellarium (optional)

Other materials (for entire activity setup):

30 m of rope or string (optional)
Sidewalk chalk or small rocks
Meter stick (you can use one of the quadrants if needed)

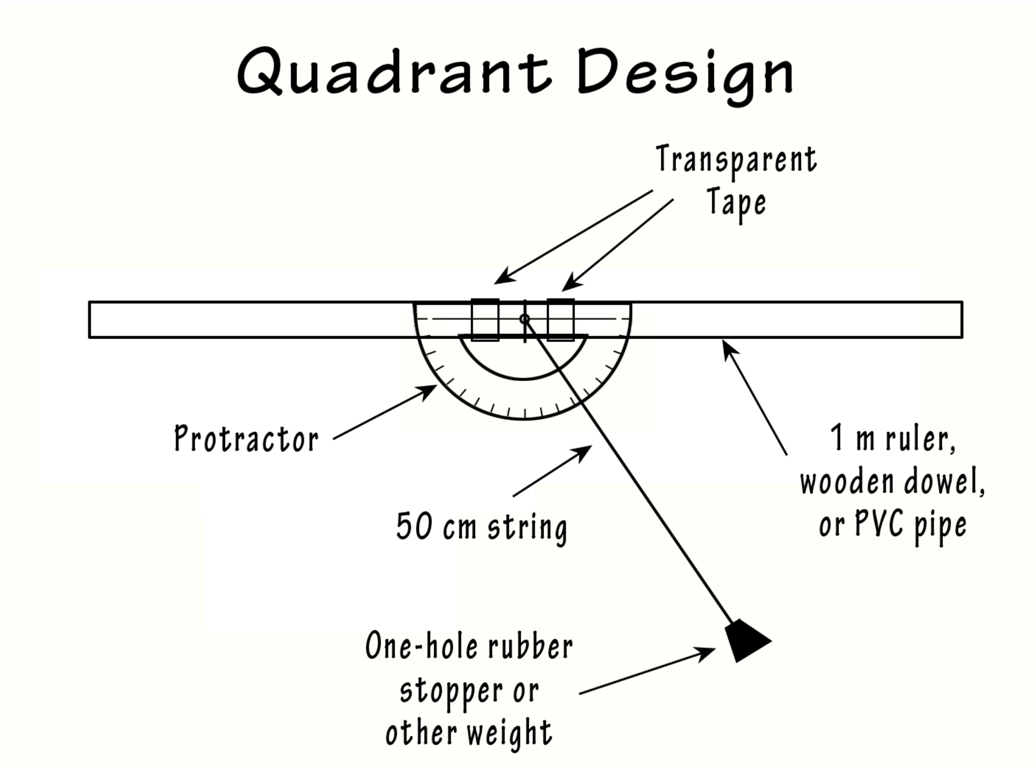
C – Teacher Set Up (about 1-2 hours):

You will need to find an area of asphalt or concrete such as a school parking lot or level grassy field (such as a football or soccer field) that won't be disturbed during the school day and is empty of cars or obstructions.

Lay out the rope or draw a straight line with chalk about 30 m long. In the center of the line, draw a circle or mark the point with rocks. This is the position of our sun. Measuring outward to the right of the center, draw circles or lay down rocks at 5 m, 10 m, and 15 m distance. To the left of center, draw circles or lay down rocks at 2 m, 7 m, and 12 m. Label the circles as Planets 1 through 6 as shown in the Parallax Diagram.

From the center of the baseline (the sun's position), measure out at a perpendicular 90 ° angle four random distances (such as 11 m, 14 m, 23 m, and 31 m) and mark them with circles or rocks also. Label them Star A through D from the sun outward. Record the actual distances to these stars to use later in the lesson.

Build a primitive quadrant for each group of four students. Use a 1 m ruler, wooden dowel, or PVC pipe as the base. As shown in the diagram, tie one end of the 50 cm string to the midpoint of the ruler or dowel, then run the unattached end of the string through the vertex hole of a protractor and tape the protractor to the ruler or dowel with the angle numbers facing in the right direction and the string hanging through the vertex hole. Then tie the other end of the string to the rubber stopper or weight. This is the plumb bob.



Part II: Engagement

A – Stereo Vision and Parallax

Ask students if they know what the term "parallax" means. Have someone look it up in a dictionary if needed. Parallax is the apparent change of position of a nearby object that is actually caused by the movement of the observer. Most students won't understand this offhand, but explain that parallax is something that you use every moment you're awake even though you might not realize it.

Ask if anyone knows why we have two eyes and two ears, besides redundancy. Explain that having two eyes allows stereoscopic (3D vision) and having two ears allows stereophonic sound (3D sound). It allows us to easily locate the position of an object or a sound in space.

B – Eye Dominance Exercise

Explain that in addition to having a dominant hand or leg, they also have a dominant eye. For one of their eyes, the finger remained on the distant line. That is the dominant eye. We really see everything as two images, but our brains automatically compose the two images into one, giving preference to whichever eye is dominant.

By controlling the images received by each eye using red-blue filters or polarization filters in eyeglasses, movie makers can create the illusion of three dimensions in films.

Explain that finding the position of distance objects such as stars can also be done using parallax, since Earth changes position every six months just like our eyes. Finding the exact position of a star is called astrometry.

Part III: Exploration

A – Measuring Parallax Angles

1. Review of right triangles and the tangent function

A right triangle has one 90° angle. The other two angles add up to 90°.

For our triangle, the baseline will be the shortest side. It represents the average distance between our sun and a planet, such as Earth or Mars. For Earth, the distance is called one astronomical unit, or A. U., and is equal to 149,598,000 km. We've labeled this as D_p in our diagram.

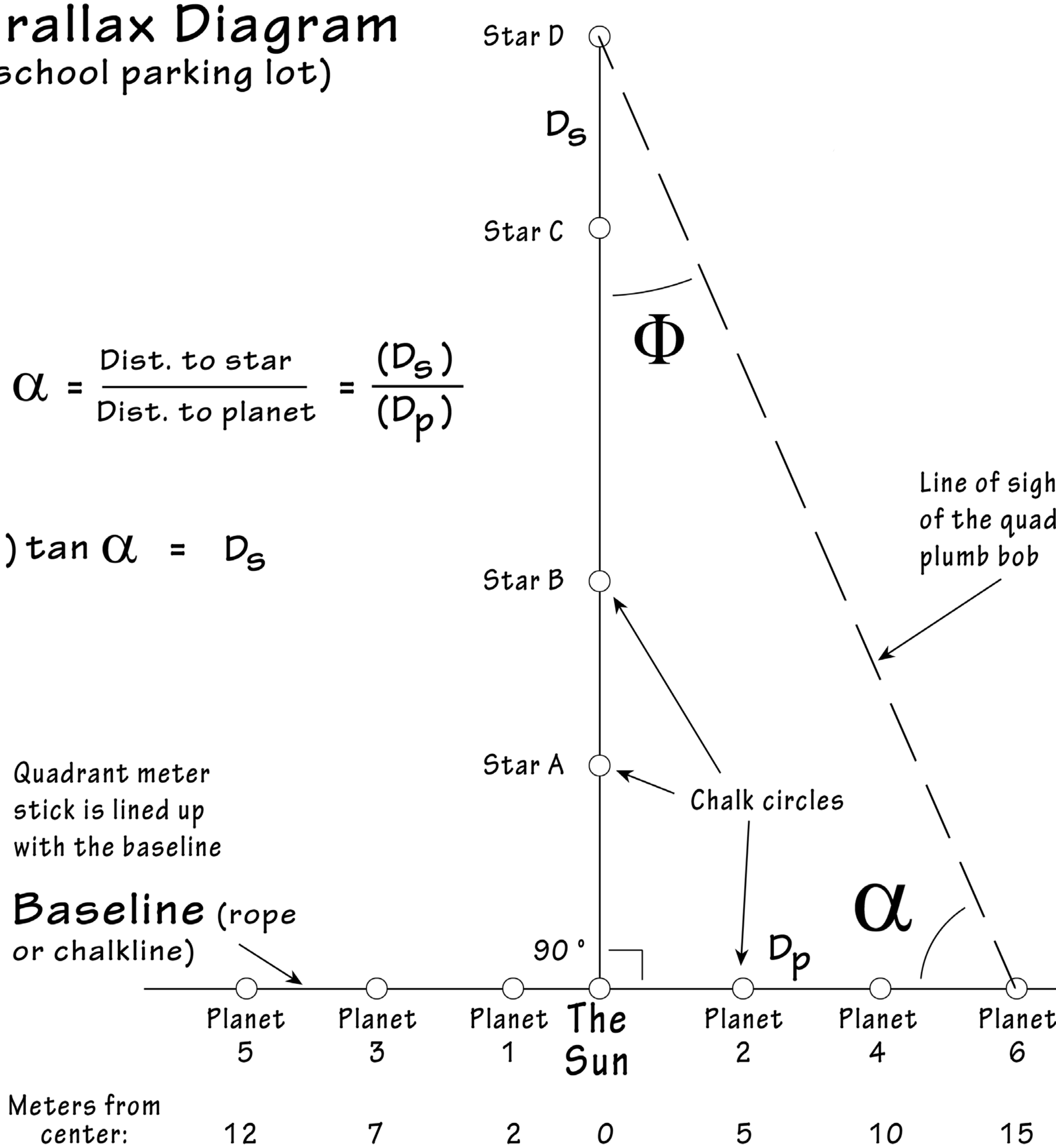
The medium side (perpendicular to the baseline) is the distance to a nearby star, which we'll call D_s.

Medieval Arabic mathematicians and land surveyors discovered relationships between the size of the angles in a right triangle and the length of the sides. This is the foundation of trigonometry. For the angle formed between our baseline and our line of sight to a star (which we'll call al-

Parallax Diagram
(in school parking lot)

$$\tan \alpha = \frac{\text{Dist. to star}}{\text{Dist. to planet}} = \frac{(D_s)}{(D_p)}$$

$$(D_p) \tan \alpha = D_s$$



pha, α), the length of the opposite side (D_s) divided by the adjacent side (baseline length or D_p) is equal to the tangent function of alpha.

Using a little cross multiplying, we find the length of the opposite side, or the distance to a nearby star (D_s), by taking the tangent of alpha and multiplying it by the planetary distance (D_p), or one A.U. for Earth. This will give the distance to the star in kilometers. We then divide that by the number of kilometers in a light year.

2. Running the activity to collect data

Divide your class into teams of four students each. Provide each team with a quadrant. If you have time, it would be good for each team to make their own quadrant.

Take the class outside to where you have marked off the planets and stars. Assign each team to one of the planets and have them use their meter rulers (or meter length dowels or PVC pipe) to measure the distance from the sun to their planet in meters. Record it in the first data table.

Have one student in the group stand at one of the stars (trade off stars between the teams) while the other three run the quadrant. They will lay the quadrant down so that the numbers on the protractor are up and the meter stick is in line with the baseline as closely as possible. Make sure they sight along the ruler in both directions to line it up.

Using the string and plumb bob, have one student sight directly to the star and line up the string and bob as closely as possible. Record the angle in the first data table. This is alpha, a. Be careful they get the angles correct – they should all be less than 90° and students often get the angles inverted (record 30° when it should be 60°, for example).

Repeat the procedure to sight in on the other three stars and have each team mark the alpha angle for all four stars from their planet's position. Each team will have different numbers since they have different baseline lengths and different parallax.

B – Back in the classroom

Create an overall data table on your white board and have each team share their own data to fill in the table.

Have each team use a calculator with trig functions to calculate the distance to each star based on their parallax angle and the baseline distance. The formula is the baseline distance multiplied by the tangent of the angle (α).

Have each team fill in their calculations for the distances. At this point, fill in the correct distances you measured as you set up, calculate the average distances calculated by all teams, and compare with the correct answer.

Part IV: Explanation

A – Discussion

Lead a discussion as to why different teams did better. If all teams were equally careful in their measurement of the parallax angles, then the following should be true:

The teams assigned planets with greater baseline distances should get more accurate results overall. Ask why (because for close planets, such as Planet 1, the angles get closer and closer to 90° and even slight inaccuracies in measuring the angle, especially with our primitive quadrants, become greatly exaggerated by the tangent function).

The further the star is, the less accurate its distance calculations will be on average. Ask why (the further the star, the closer the angle is to 90° and the less accurate the tangent function will be).

In actuality, astronomers use the angle that a star appears to move back and forth compared to more distant background stars as the Earth orbits the sun, which is labeled phi (ϕ) on our diagram.

The parallax shift is measured six months apart to get the greatest angle, then divided by two to get the angle.

The formula in this case is D_p (now the opposite side) divided by D_s (now the adjacent side) equals the tangent of phi (ϕ).

B – Additional facts

1. The distance of Earth to the sun (one AU) is very small compared to interstellar distances, so parallax angles are smaller than one arcsecond, which is 1/3600 of a degree.

2. An alternative distance measurement for stars used by most astronomers is the parsec. A star with a parallax angle of 1 arcsecond has a distance of 1 parsec, or 1 parsec per arcsecond of parallax, which is about 3.26 light years. The closest star system to us is Alpha Centauri, which has a parallax angle of .742 arcseconds, or a distance of 1.35 parsecs. At 3.26 light years per parsec, that makes a distance of about 4.4 light years to Alpha Centauri A and B.

3. For stars beyond about 200 light years, the parallax angle is so small (< .01 arcseconds) that parallax can no longer be used to find the distance to a star with an error of less than 10% using ground-based measurements. Other techniques must be used (see Extension Activity D) beyond 200 light years.

4. The Hipparcos satellite (1989-93) measured the positions of stars with great accuracy, since it was above the fluctuating atmosphere. It was able to measure parallax angles to over 118,000 stars with an unprecedented accuracy of 1 milliarcseconds (.001 arcseconds) and 1 million other stars at 25 milliarcsecond (.025) accuracy (which is still better than anything that can be done from the ground). It has extended the distance determinations for which parallax can be used to about 1000 light years.

C – Questions

Have the students answer the following questions on their own as homework:

1. On average, which planets were more accurate from which to measure the distance of a star, planets that are close to the sun or farther away? Why?

2. On average, which stars are more accurate to measure distances for, close stars or farther stars? Why?

3. How would you modify the formula if you were to use the actual parallax angle of phi (ϕ) as astronomers do instead of the alpha (α) angle we used?

4. For an actual star with a parallax angle of .19444 arcseconds, what is its distance from us in kilometers? Use 149,598,000 km as the distance from the Earth to the sun. Remember that one arcsecond is 1/3600 of a degree.

Part V: Elaboration and Extension Activities

Assign individual students (or small groups) to choose one of the following activities to extend the discussion and to write a brief report or journal entry on their research or results, then share with the rest of the class the next day.

A – Astronomy Software

Use planetarium software such as Stellarium, Distant Suns, or other programs to look up several of the following nearby stars: Alpha Centauri, Tau Ceti, Epsilon Indi, Epsilon Eridani, Procyon, Sirius, Vega, or Altair. Look up the parallax angles listed and use the tangent function to calculate the distances. Keep in mind we are using the actual parallax angle, or phi (ϕ) and not alpha, so the formula you must use divides the tangent of phi into the planetary distance (1 AU) to get the stellar distance.

For example, Sirius has a parallax angle of .37921 arcseconds. Divide that by 3600, and you have the parallax angle in degrees (.00010534 °). Take the tangent function, and you get .0000018385. Then divide that into 1 AU (149,598,000 km) and you get about 81.37 trillion km as the distance to Sirius. Of course, that's not a very usable number, so let's calculate the distance in light years. Light travels 299,792 km/second. Multiply that by 60 to get minutes, by 60 again to get hours, by 24 to get days, and by 365.26 days to get years. That's about 9.46 trillion km that light travels in a year, or 1 light year = 9.46 trillion km. Divide that into our 81.37 trillion km distance to Sirius, and we get (viola!): the distance to Sirius is 8.6 light years. This may seem complicated, but this is exactly how the distance in light years is calculated for stars.

Repeat this exercise for about five stars and compare your calculations with the ones listed in the software. They should agree very closely.

B – Land Surveying

Have a group of students research the history of surveying land. How do surveyors measure the height of a distant mountain if they know how far away the mountain is? What is a surveyor's transit, and how does it work? What are some of the previous instruments used?

C – Solar Navigation

Before modern GPS satellites and other systems, how did ship captains and navigators measure their position (latitude and longitude) at sea? What devices have been used over the centuries to do this? Why is the North Star (Polaris) essential for calculating latitude, and how is the sun's position above the horizon (its azimuth) at a particular time of day and the day of the year essential to finding longitude? What other techniques are used for longitude measurement, and how was this problem first solved?

You may also have them read about the Imperial Trans-Antarctic Expedition of the H.M.S. Endurance led by Ernest Shackleton and their perilous crossing from Elephant Island to South Georgia Island in a small lifeboat, traveling over 1200 km across open ocean for 15 days in stormy weather using only a few sun sightings by Captain Frank Worsely.

D – Ladder of Cosmological Distances

Since parallax can't be used beyond about 2000 light years, even using Hipparcos data, what other techniques are used? Have students look up the concepts of standard candles used to measure distances, such as the distance modulus formula comparing absolute and apparent magnitudes of stars, the use of Cepheid variable stars and Type 1a Supernovas for further distances (such as nearby galaxies) and red shift for distant galaxies. How does all of this add up to a "ladder" of cosmological distances?

E – 3D Imagery

Have a group of students look up the techniques used to create the illusion of three dimensions in motion pictures. How do the glasses separate out the two image streams, and how are the streams filmed and projected? What are the technologies involved? How do red-blue anaglyphs work, and how are they used for navigating robotic rovers such as the Mars Science Lab (Curiosity) on the surface of Mars?

F – Right Ascension and Declination

The distance to a star is only one of three polar coordinates used to locate a star in three dimensional space. What are the other two coordinates, which correspond to celestial latitude and longitude? How are they measured, and what are the starting reference points? Have the group develop a 3D model of a constellation, such as Orion, by looking up the coordinates of the major stars, then creating a scale and hanging beads tied to thread from a piece of cardboard in correct relationship to each other in 3D space. Demonstrate how the constellation only looks as it does from Earth from only one angle, but changes dramatically if viewed from any other vantage point.

Part VII: Evaluation

A – Formative:

Successful completion of the activity (measuring the parallax angle and calculating the distances to the stars), as well as participating in the discussion after and answering the homework questions.

B – Summative:

A practical exercise to test how well the students can generalize and apply their knowledge would be to have them calculate the height of a local landmark that is visible from their school. Have them use Google Earth's ruler feature to find the distance between your school and the landmark. Then have the students use a quadrant on an area of level ground and measure the angle to the top of the landmark, then use the tangent function correctly to solve for the landmark's height. If possible, look up the actual height of the landmark. If you live near hills or mountains, they will do. A tall building or even a water tower or cell phone tower will do as long as they are near enough and tall enough for the height angle to be measured.

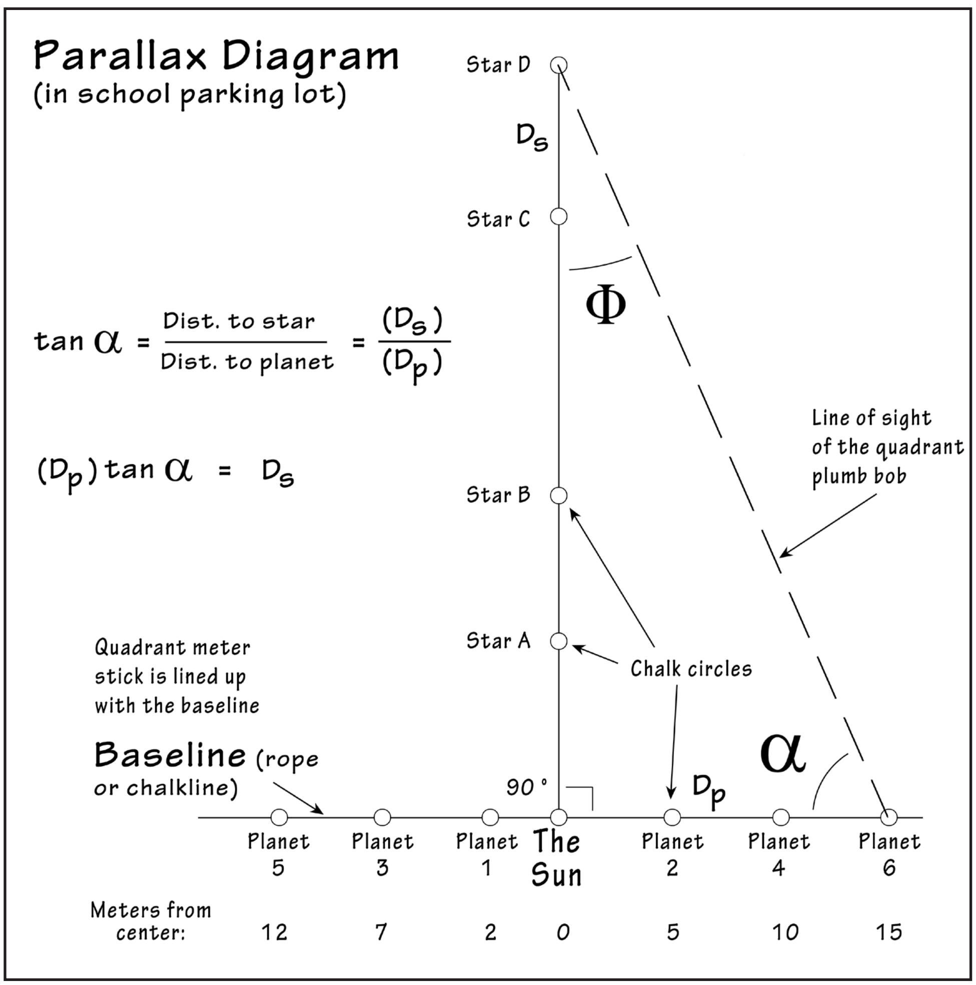
Student Data Sheet

Name: _________________________________ Date: ___________ Period: ____

Instructions: Fill in the following information for each of planets in the diagram your teacher has laid out. Use your meter ruler to measure the distance from the sun to your planet in meters, then use the quadrant to measure the angle between the baseline (between the sun and your planet) and each of the stars. Make sure your ruler is lined up perfectly with the baseline and that you sight along your string and bob to each star as accurately as you can.

Back in the classroom, share data with the other teams so that you have the distances from the sun to each planet and all of the angles from each planet to each star. Use a calculator with trigonometry functions to calculate the distance to each star as measured from each planet, based on the formulas shown in the diagram below. Fill in the overall data table on the next page with your calculations.

Planet No.	Planet Distance	Angles to Stars			
		Star A	Star B	Star C	Star D



Calculated Distances as Measured from Each Planet:

	Calculated Distances to Stars from Sun			
	Star A	Star B	Star C	Star D
Planet 1				
Planet 2				
Planet 3				
Planet 4				
Planet 5				
Planet 6				
Average				

Actual Star Distances:

To calculate the percentage errors, take the difference between actual distance and the calculated average distance from above, then divide that difference by the actual distance and multiply it by 100 to convert to percent. For example, if the actual distance to a star is 12 m but the calculated average distance as measured from all the planets is 12.7 meters, then the difference is .7 m. .7 m divided by 12.0 m times 100 gives 5.8% error.

	Average Distances	Actual Distances	Percent Error
Star A			
Star B			
Star C			
Star D			

Answer the following questions based on your data and the discussion in class:

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