

SOLUTION USING SIMPLEX METHOD

If you would like to further talk about the solution methods in the classroom, you could show how to solve this problem using simplex method as follows:

In our diet problem, we are looking at a “standard” minimization problem. What does the “standard minimization” mean? It means that the constraints are all linear; the constants are non-negative; the inequality symbols are greater or equal to; the non-negativity constraints are placed on our variables; and, we’re minimizing. After recognizing that it’s a standard minimization problem, in order to apply the simplex method, we need to convert it to a standard maximization problem. To do that, we will start writing a matrix, which comes from the coefficients and constants of the constraints as well as the coefficients and constant of the objective.

$$A = \begin{bmatrix} 12 & 33 & 135.5 \\ 3 & 6 & 45.5 \\ 9 & 1 & 32.5 \\ 171 & 150 & 1,000 \\ 2 & 1.5 & 0 \end{bmatrix}$$

Then, we will find the transpose of this matrix by interchanging its rows and columns.

$$A^T = \begin{bmatrix} 12 & 3 & 9 & 171 & 2 \\ 33 & 6 & 1 & 150 & 1.5 \\ 135.5 & 45.5 & 32.5 & 1,000 & 0 \end{bmatrix}$$

We can see that the rows of this matrix are the columns of the first matrix. This new matrix is a maximization problem so that the corresponding maximization problem is:

$$\text{Maximize } w = 135.5 y_1 + 45.5 y_2 + 32.5 y_3 + 1,000 y_4$$

subject to the constraints

$$12 y_1 + 3 y_2 + 9 y_3 + 171 y_4 \leq 2$$

$$33 y_1 + 6 y_2 + y_3 + 150 y_4 \leq 1.5$$

$$y_1 > 0, y_2 \geq 0, y_3 \geq 0, \text{ and } y_4 \geq 0$$

This formulation is called the dual of the original problem. Since this is the standard form for a maximization problem, we can now apply the simplex method. To do that, the first step is to

Step 1: Rewrite the objective formula as an equation: $w - 135.5 y_1 - 45.5 y_2 - 32.5 y_3 - 1,000 y_4 = 0$. We will call this the objective equation.

Step 2: Remove the inequalities from the constraints. To do that, we will introduce two slack variables as m and n and add them to the constraints. We will call these the constraint equations:

$$w - 135.5 y_1 + 45.5 y_2 + 32.5 y_3 + 1,000 y_4 = 0 \rightarrow \text{objective equation}$$

$$12 y_1 + 3 y_2 + 9 y_3 + 171 y_4 + m = 2 \rightarrow \text{constraint equation}$$

$$33 y_1 + 6 y_2 + y_3 + 150 y_4 + n = 1.5 \rightarrow \text{constraint equation}$$

Step 3: Let's next put all of these in a tableau.

y_1	y_2	y_3	y_4	m	n	w	RHS values
12	3	9	171	1	0	0	2
33	6	1	150	0	1	0	1.5
-135.5	-45.5	-32.5	-1,000	0	0	1	0

RHS = right hand side

Step 4: Our next step is to identify the pivot. To do that, we first find the pivot column. The pivot column is the column with the largest negative value in the objective equation. Let's draw a line with the green marker around the pivot column. Next comes identifying the pivot row. This is done by dividing the values of the constraints by the value in the pivot column. The lowest value gives the pivot row. 2 divided by 171 = 0.012; 1.5 divided by 150 = 0.01. The pivot is where the pivot column and pivot row intersect. Let's draw a line around the pivot with the red marker.

y_1	y_2	y_3	y_4	m	n	w	RHS values	Ratios
12	3	9	171	1	0	0	2	2/171
33	6	1	150	0	1	0	1.5	1.5/150
-135.5	-45.5	-32.5	-1,000	0	0	1	0	

Step 5: We now need to make the pivot equal to zero. To do this, we divide the pivot row by the pivot value. It's best to keep the values as fractions.

y_1	y_2	y_3	y_4	m	n	w	RHS values
12	3	9	171	1	0	0	2
33/150	6/150	1/150	1	0	1/150	0	1.5/150
-135.5	-45.5	-32.5	-1,000	0	0	1	0

Step 6: Next we need to make the pivot column values into zeros. To do this, we add or subtract multiples of the pivot column. The pivot column value in the other constraint will go to zero if we add -171 times the pivot row. The pivot column in the objective function will go to zero if we add 1,000 times the pivot row. This then is the first iteration of the simplex method.

y_1	y_2	y_3	y_4	m	n	w	RHS values
-25.6	-3.8	7.9	0	1	-1.1	0	0.3
0.2	0.04	0.007	1	0	0.007	0	0.01
-25.5	-25.5	-29.2	0	0	3.3	1	5

Step 7: Finally, we will check to see if any values in the objective equation are negative. Here we have -25.5 and -29.2, so we're not quite done yet! We go back to step 4 and repeat the same procedure. After performing steps 4 through 7 two more times, we will end up with no negative values in the objective equation. Here is how the final tableau looks like.

y_1	y_2	y_3	y_4	m	n	w	RHS values
-0.5	0	1	11.3	0.12	-0.06	0	0.15
5.6	1	0	23.1	-0.02	0.18	0	0.23
101.6	0	0	918.9	2.93	6.12	1	15.04

We now have to determine the values of the variables that give the optimum solution. Remember we used the dual to solve this. What that means is that we cannot pull the final solution from the tableau in the same manner as we were to apply the simplex directly to a standard maximization problem. In the dual problem, the slack variables give us the solution to the original minimization problem: $m = 2.9$ and $n = 6.1$. The optimal value for the original is the same as the optimal value for the dual problem. Therefore, the solution to our original problem is $z = 15.04$ with $x_1 = 2.93$ and $x_2 = 6.12$