Flaws of Averages

BLOSSOMS Module

Supplement to Flaw of Averages #1:

"The average is not always a good description of the actual situation"

Contents:

We hope that our opening examples of Rhonda with the average colored outfit, Dan with the average location, and then the crossing of the river were clear for an introduction to our first flaw of averages: "the average is not always a good description of the actual situation."

Here we introduce a few other simple examples and a more advanced topic of this flaw of averages that you may find helpful in your discussion with your students after this opening segment.

Average Age

Thanks to a discussion with one of our colleagues, the following example puts a family touch on this flaw of averages.

Imagine the following scene: a 91 year-old woman is sitting in a chair, holding her 1 year-old great-granddaughter on her lap. In this scene, the average age of the two of them is 46. Saying that 'there are two people in this scene with an average age of 46' is clearly a poor representation of the actual scene in this case.

Average Length of Sticks

Similar to the average age example, another simple example of this flaw of averages would be to bring to class two sticks (or pieces of wood, or anything with an obvious 'length') of widely varying length, i.e. a stick that is 2 meters long and a stick that is 2 centimeters long.

The average length of the two sticks is 1.01 meters, and saying that 'there are two sticks with an average length of 1.01 meters' poorly reflects the actual situation with a long and very short stick.

Average Wealth in a Restaurant

Imagine that a woman with one billion dollars walks into a restaurant with 50 guests already seated. Assuming that everyone seated in the restaurant has a non-negative amount of money, this woman's arrival means that everyone **on average** is suddenly at least a multi-millionaire!

Advanced Topic: Average of Continuous Distributions

Note: This example is intended for advanced high school students that are familiar with the concepts of random variables, continuous distributions and the process of finding an expected value of a continuous function. Online references to these (and other) topics are included on the Flaws of Averages website.

Our terminology here is as follows:

X : random variable with a continuous probability density function

$f_X(x)$: probability density function of random variable X

$$E[X] = \int_{-\infty}^{+\infty} \left(x * f_X(x)\right) dx \text{ : expected value of X}$$

Let X_1 be a random variable with a normal probability density function where the mean is 1 and standard deviation is also 1:

$$f_{X_1}(x_1) = \frac{1}{\sqrt{2\pi}} e^{-x_1^2/2} \qquad -\infty < x_1 < +\infty$$

$$E[X_1] = \int_{-\infty}^{+\infty} \left(x_1 * \frac{1}{\sqrt{2\pi}} e^{-(x_1-1)^2/2} \right) dx_1 = 1$$

Let X_2 be a random variable with an exponential probability density function with a parameter equal to 1:

$$f_{X_2}(x_2) \quad = \quad \begin{cases} e^{-x_2} & x_2 \geq 0 \\ 0 & otherwise \end{cases}$$

$$E[X_2] \quad = \quad \int_{-\infty}^{+\infty} \left(x_2 * e^{-x_2}\right) dx_2 \quad = \quad 1$$

In both cases, the average value of the continuous distributions, $E[X_1]$ and $E[X_2]$, are equal to 1. However, as the image below helps illustrate, the range of values that the random variables X_1 and X_2 could take on are clearly different. Hence, simply saying that the average value of a continuous random variable is 1 without mentioning the shape of the continuous distribution as well is also a poor description of the actual situation.