Video on Blossoms/Quadratics

Gilbert Strang, MIT
gilstrang@gmail.com

Notes on the questions suggested before video pauses

After part 1: The parabola \( y = -x^2 + 4x + 9 \) opens downward because of \( -x^2 \). Since \( y = 9 \) at \( x = 0 \), the parabola must cross the \( x \) axis twice – before and after \( x = 0 \). The formula for the roots produces \( 2 \pm \sqrt{13} \), giving practice with square roots.

After part 2: The parabola \( y = x^2 - x \) factors into \( x(x - 1) \). Then \( y = 0 \) when \( x = 0 \) and also when \( x = 1 \) (two real roots).
   The parabolas \( y = x^2 - x + 1 \) and \( y = x^2 - x + 2 \) have no real roots. You can see that \( x^2 - x + 1 = 0 \) is impossible since \( \text{all real } x \text{ are smaller than } x^2 + 1 \). (Why? Because if \( |x| > 1 \) then \( x^2 \) is bigger, and if \( |x| < 1 \) then \( 1 \) is bigger.) The quadratic formula gives the complex roots of parabolas:
   \[
y = \frac{1 \pm \sqrt{-3}}{2} \quad \text{and} \quad y = \frac{1 \pm \sqrt{-7}}{2}.
   \]

After part 3: The area with sides \( x \) and \( 50 - x \) is \( y = -x^2 + 50x \) with slope \( -2x + 50 \). Then the area is maximum where the slope is zero, at the top of the parabola. This top point has \( x = 25 \) and the rectangle is actually a square.

After part 4: A square is the best of all rectangles but not the best of all shapes. \textit{That honor goes to a circle.} The circumference going around the circle is \( 2\pi r \) and the area is \( \pi r^2 \):
   \[
   2\pi r = 100 \text{ meters and } \pi r^2 = \pi \left( \frac{100}{2\pi} \right)^2 = \frac{2500}{\pi} \approx 795.
   \]
   This circular area of 795 easily defeats the rectangular area of \( (25)^2 = 625 \).

Question for superstars about two parabolas \( y = ax^2 + bx + c \) and \( Y = Ax^2 + Bx + C \). What condition on the six numbers \( a, b, c, A, B, C \) means that the parabolas never touch?