## Teachers Guide

Dear teacher.
In this lesson we shall discuss some topics in graph theory. This lesson was designed to be different from what students have learnt in school specially algebra, geometry and arithmetic.

The goal of this lesson is to raise student's level of thinking and to show them some applications of mathematics. This may motivate them to learn and enjoy learning mathematics.

We shall discuss the following concepts:

- Properties of planar graphs
- Euler's formula which relates the number of vertices, edges and faces in a planar graph
- The four color theorem which says that 4 colors are sufficient to color the faces of any plane map in such a way adjacent faces receive different colors.
- How to avoid crossings in electrical circuits as an application of graph theory.

Through this lesson, students are expected to do some in class activities. At the beginning of the class you may divide students into groups of 3 to 5 students each. This is a summary of activities that students are asked to do.

Activity -1: ( 2 minutes) Ask 3 students to represent the stations and another 3 to represent the houses. The rest of the class may try to link each station to each house through ropes or wires with different colors; one color for water, one for gas and one for electricity.


Activity -2: ( 2 minutes) we ask each student to draw a graph with 5 vertices and 10 edges so that no more than one edge connects two different vertices. Compare the graph with his colleagues in the group to see if they got the same graph.

Activity -3: ( 3 minutes) we ask the students to draw the graphs ${ }_{5} \mathrm{~K},{ }_{4} \mathrm{~K},{ }_{3} \mathrm{~K},{ }_{2} \mathrm{~K},{ }_{1} \mathrm{~K}$ and the complete bipartite graphs ${ }_{3,3} \mathrm{~K},{ }_{2,2} \mathrm{~K},{ }_{1,2} \mathrm{~K},{ }_{1,1} \mathrm{~K}$; discuss with the group which of these can be drawn on the paper with the edges crossing each other.

Activity -4: ( 3 minutes) students will be given a sheet with a graph (b) drawn on it and we ask them to cut the paper along the edges of the graph. We ask them to see if there is a relation between the number of vertices, edges and pieces. Discuss your finding with your friends in the group.


Activity -5: ( 3 minutes) we ask students to see if we can apply Euler's formula on graphs drawn on the sphere. Try to find a relation between the number of vertices, edges and faces for graphs on the sphere and on the tube.

Activity -6: ( 2 minutes) we distribute papers with the map of the regions in the kingdom and we ask them to color the map with minimum number of colors. What is this minimum number? Is it the same for all plane graphs?


Activity -7: ( 3 minutes) we distribute carton boards and wires (or threads) and ask them to connect the vertices as shown on the screen without the wires crossing each other. If they cannot do that, let them think of a way to connect the vertices without edge crossing each other. What method will they use? Will this lead to see how scientists resolve the problem of edge crossing on the mother board? (


## Sketch of the motherboard

As on extra work for good students, we may ask them to prove Euler's formula for plane graphs using mathematical induction on the number of edges. We may remind them with the 3 steps of math induction and ask them to prove the formula.

Thank you very much.

