Blossoms Monty Hall Problem

[00:00:00.00]
[00:00:00.99] [MUSIC PLAYING]
[00:00:20.79] [PHONE RINGING]
[00:00:24.76] SAM: Hello.
[00:00:25.60] JOHN MACINTOSH: Hello, Sam. This is John Macintosh, host of the Monty Hall Game Show and I'm here with your friend, Cameron. Now Cameron here has a chance to win a new car, but he needs your help deciding what to do. That's why he asked me to give you a call.
[00:00:37.02] SAM: I'll certainly do my best to help out.
[00:00:39.15] JOHN MACINTOSH: Great. Now here's the situation. Cameron is faced with three doors. Behind two of those doors are goats, and behind the other door is a brand new car. Only two people in the universe know the correct location of the prize, my assistant and I. All Cameron need to do is pick the correct door and he'll walk away with a prize. Now here's Cameron with the rest of the details.
[00:00:58.72] CAMERON: Hey Sam, how's it going? So I initially picked one of the three doors. Then the assistant opened one of the two remaining doors revealing a goat. Now I have two options. I can switch to the other remaining door or I can keep my original door. I have no idea what to do here so I thought I'd ask you for help.
[00:01:15.84] SAM: I'll do my best, but I'm not sure either, Cameron. In fact, I can use some help. Can you guys discuss this with your neighbors and your teacher, try to figure out whether or not Cameron should switch doors, or if it doesn't matter? Then get back to me a few minutes.
[00:01:41.62] CAMERON: So you're saying I should switch? And that if I do switch, I'll have a 2/3 probability of winning? I'm not really sure how that works, but I'll trust you. John, I'd like to switch doors.
[00:01:51.37] John MACINTOSH: All right, Cameron.
[00:01:51.97] Voice over guy: A new car!
[00:01:53.23] CAMERON: I won! I won! I can't believe I won! Woo! So Sam, how did you know that by switching I would have a 2/3 probability of winning?
[00:01:59.48] SAM: Well I wasn't sure at first, but then I made a chart of all the different possible outcomes of the game and how they depended on the different scenarios. So beginning with which door the host placed the car behind, which door you chose, which door the assistant revealed, and then what the outcome would be if you switched or didn't switch. Being able to

visualize all the different possible scenarios really helped. But my real breakthrough came in recognizing the trick to the game.
[00:02:28.55] CAMERON: And what's that?
[00:02:29.65] SAM: Well I figured that the assistant would never open the door with the car behind it because then the show would be over and you know, the game wouldn't be any fun.
[00:02:38.19] CAMERON: Oh, that makes sense. But the guy before me went and he switched doors and he still lost. How did that work?
[00:02:44.88] SAM: Well he made the right decision. Switching doors give you a 2/3 probability of winning, but that also means that there's a 1/3 probability that you'll switch doors and still lose. And that person fell in the unlucky third. Anyways, the probability of 2/3 really pays off when you have a large number of games. It means that if you were to play 100 times and switch doors every time, that you'd win approximately 2/3 of the time. But if you only play once, you may still lose even though the probability is in your favor.
[00:03:16.19] CAMERON: Oh I see. So even though he made the decision that gave him the highest probability of winning, because it's just a probability, he still stood a chance to lose.
[00:03:23.81] SAM: Yeah.
[00:03:24.76] CAMERON: But if I took everyone that had ever been on the game show, about 2/3 of the people who switched would've won?
[00:03:30.25] SAM: Yeah.
[00:03:31.30] CAMERON: OK. That makes sense.
[00:03:32.20] SAM: But you don't have to take our word for it. You can play the game and see for yourself.
[00:03:36.60] CAMERON: Have your teacher pass out envelops, stones, and candy and play the Monty Hall game with a partner and see if switching really is the best decision.
[00:03:55.95] Hi everyone. I hope your game went well. How many people who switched ended up winning?
[00:03:59.96] SAM: Now that you guys have played the game for yourself, let's see how I was able to correctly determine that switching gives a 2/3 probability of winning. I used something called a decision tree to visualize all the different possible outcomes of the game and to determine what the best decision would be. Using a simple example, we'll show you guys how to make a decision tree, what the parts are, and then how to use it to correctly determine what the best decision should be.

[00:04:24.72] CAMERON: Decision tree is a tool to help make decisions when there are uncertainties related to different outcomes.
[00:04:29.43] SAM: Consider, for example, a decision many of you and your parents may make every morning, whether to drive or take the bus to work or school. Which is the better option if you want to get there in the least amount of time? A decision tree gives us a quantitative method for choosing between these two or more alternatives. The first step we take is to write down the decisions to be made and the possible choices.
[00:04:54.88] CAMERON: Notice how Sam drew one arrow for each possible choice. In this case, there are only two choices, but if there were more, you could just add additional arrows. This section of the tree is called the decision makers choice.
[00:05:08.10] SAM: Once I've identified the possible decisions, I now need to think about the possible outcomes each decision could lead to. While driving, I could either hit light or heavy traffic. And the bus could be running either very frequently or just normally. Every morning, I tune in to the radio and I listen to the traffic report and the weather. Based on both of these, I have to estimate how long it would take to drive in versus how long it would to take the bus.
[00:05:36.95] One morning, I estimate that there's a 70% probability the bus runs every two to five minutes and that there's a 40% probability of hitting light traffic if I drive. I'll write these probabilities next to the arrows for each of the possible scenarios.
[00:05:54.16] CAMERON: And that segment of the decision tree is called nature's choices where there are multiple possible outcomes, each with a known or estimated probability. Next, we finish making this decision tree by drawing the endpoints.
[00:06:06.52] SAM: The endpoints are where we write down what will happen in each of these four possible scenarios. From experience, I know that driving in light traffic takes only three to five minutes. But if there are traffic jams, it'll take 20 to 25 minutes. I also know that the bus ride takes only five minutes. So if the bus is running every two to five minutes, then I can arrive at work in seven to 10 minutes. But if the bus runs only every 10 to 15 minutes, then it'll take between 15 and 20 minutes to arrive.
[00:06:36.42] CAMERON: So now we have a finished decision tree and it allows us to visualize graphically all the possible ways our trip could turn out before we even leave. We can see that one possibility is driving in traffic jams in which case it takes 20 to 25 minutes to get to work, or we could take the bus, and depending on how frequently it's running, arrive in between 7 to

20 minutes. Now the last step is to use the tree to calculate something called the expected value.
[00:07:01.12] SAM: The expected value or average value is just what you expect to have happen as a consequence of your decision. In our case, it's the time it takes to get to school or work when you're driving or when you're taking the bus.
[00:07:15.77] CAMERON: The way you calculate the expected value for a given decision is to multiply the probability of each branch along the pathway to its endpoint, and then to sum the results up over all pathways within that decision. For example, to calculate the expected value of the commute time for the decision to drive, you multiply the time it takes to arrive in light traffic, five minutes, by the probability of light traffic, 0.4, and add that to the time it takes to arrive in traffic jams, 25 minutes, by the probability of traffic jams, 0.6. When we add the two together, we obtain 17, meaning that the expected value of the time to drive to work is 17 minutes.
[00:07:53.14] SAM: We can then do the same thing for taking the bus, multiplying the probability of the bus running every two to five minutes with the time it takes to arrive, and adding that the probability of the bus running every 10 to 15 minutes multiplied by the time it would take to arrive. When we do this, we obtain an expected value of 13 minutes, meaning that if you take the bus, you can expect to have a commute time of 13 minutes.
[00:08:20.07] CAMERON: Now that we have the two expected values of our decision, we see which is the better choice. In this case, taking the bus gives the better expected value, a shorter commute, and that is our decision.
[00:08:30.08] SAM: By making a decision tree, we were able to systematically list all the possible outcomes of this choice and characterize how they depend on the underlying probabilities. This is the exact same thing I did to figure out the best decision in the Monty Hall problem. The only difference was that the decision tree for the Monty Hall problem was a little more complex because there were three levels instead of two, but the procedure was the same-- list all the choices, determine the probabilities and endpoints, and then calculate the expected value to determine the best decision.
[00:09:08.14] CAMERON: Now that you guys have seen how to make and use a decision tree, get back with your neighbor and work on making the decision tree for the Monty Hall problem. Here's a hint, the Monty Hall problem has four different steps. First, the game show host picking a door to place the car behind. Second, the contestant initially choosing their door. Third, the assistant opening a door to reveal a goat. And fourth, the contest

having an opportunity to switch. Each of these steps should have its own branch in the decision tree.

[00:09:44.31] SAM: We hope you had fun with the exercise and learned from it. I'll bet that many of you found a correct decision tree. What if, for example, you had 26 doors, 25 goats, and one car? What would be the probability of winning the car if you switch doors after 24 goats had been revealed? While you're thinking about it, why not try it out with a live demonstration? Ask your teacher to please pass out the envelopes, candy, and stones and give it a try it for yourself.

[00:10:24.49] CAMERON: Now we'll go through the decision tree for the 26 door version of the Monty Hall problem. However, we'll show a slightly different spin on the decision that we did with the three door version.

[00:10:34.19] SAM: With 26 doors it would be a pain to draw out all the different possible paths. Instead, we can think about the problem differently and separate it into one decision maker's choice and one nature's choice.

[00:10:48.29] CAMERON: In this case, the decision maker's choice is whether or not to switch an envelope, and the two possible choices are yes or no.

[00:10:55.00] SAM: The next part is nature's choice, and the two possible scenarios are that you initially pick the prize or you picked nothing. In this case, with 26 envelopes and one prize, the probability of picking the prize is 1/26, and the probability of picking nothing is 25/26.

[00:11:15.63] CAMERON: Now we write down the outcomes for each of these four scenarios-- switching and initially picking the prize, switching and picking an empty envelope, not switching and picking the prize, and not switching and picking an empty envelope.

[00:11:29.98] SAM: From this, we see that if you switch, you will win when you initially picked nothing. We can calculate the expected value and see that it is 25/26. On the other hand, we calculate the expected value and find that it is 1/26. Therefore, switching will win you the prize 25 out of 26 times, approximately.

[00:11:53.30] CAMERON: By thinking about the problem this way, we saved having to draw out every single outcome, but we also saw how thinking about what are called limiting cases can help build up your intuition.

[00:12:04.30] SAM: In the case where there are 26 envelopes and only one prize, it's easy to see that the probability of picking the prize initially is very small. Then after 24 envelopes had been opened, you know that the prize is either in the envelope you initially chose or the one envelope left unopened by the assistant. Since the probability you initially picked the envelope with

the price is very small, the probability it is in the remaining envelope is very large. Therefore, if you switch, then you have a large probability of winning. And of course, again, the decision tree confirms this newly found intuition.
[00:12:45.37] CAMERON: That makes a lot of sense, but one thing I'm still curious about is if you decide to switch and still lose, does that mean you made a bad decision?
[00:12:53.44] SAM: I don't think so. And here's an example I think which show us why. Suppose I listen to the weather one morning and I hear it's going to rain so I decide to bring my umbrella and I carry it with me all day but then it doesn't rain a drop. I didn't make a bad choice and I'm not mad that I brought my umbrella because it's still offered me protection in case it was going to rain.
[00:13:12.07] CAMERON: Hmm. I think I thought of an example. So for my new car, I probably have to buy car insurance. Every month I'll pay a premium, and that way if I get into an accident, it'll be covered. But if I don't get into an accident, I'll still be happy I made the purchase because I'll get the peace of mind that comes with the coverage. And speaking of cars, I think I left the windows open.
[00:13:30.32] SAM: While we're checking on the car windows, why don't you guys talk to your neighbor about some times in your lives when you could follow the optimal decision but still not get the optimal outcome. Think it over and we'll meet you back at the car.
[00:13:53.25] CAMERON: So today we learned how to help your friends win a car, but we also learned how to use decision trees to make smart choices about probabilistic or uncertain events.
[00:14:01.03] SAM: We saw how decision trees let you visualize all the different possible outcomes, calculate the expected values, and make smarter choices.
[00:14:09.73] CAMERON: So we also tried to show you how these skills apply to more than just game shows. They help with everyday life choices from what rain gear to wear to how to get to school in the morning.
[00:14:17.91] SAM: So we hope we've given you guys a small appreciation for the power of probability and also some tools to help you and your friends make smarter decisions.
[00:14:28.88] CAMERON: Thanks for letting us into your classroom today and helping me win this brand new car!
[00:14:31.85] [ENGINE STARTING]
[00:00:00.00]
[00:14:49.56] CAMERON: Hello and thank you for your interest in our Blossoms presentation on the Monty Hall problem and probability.

[00:14:54.88] SAM: Our learning objectives in this lesson is to teach your students about decision trees and how they can be used to help make smart decisions about uncertain or probabilistic events.
[00:15:05.14] CAMERON: There are no major prerequisites to our lesson, although the students should have a grasp of probability to the extent that they can figure out the choice between two goats and a car.
[00:15:13.89] SAM: Over the course of our lesson, your students will participate in several activities. The first is to talk with their neighbors or in groups about whether or not Cameron should switch doors in his game show scenario.
[00:15:26.33] CAMERON: The second activity is to recreate the Monty Hall game show in class, having the students and their neighbors work together. One should play the host and one should play the contestant. The third activity is to make the decision tree for the three door scenario.
[00:15:39.56] SAM: The fourth activity is to play the 26 door Monty Hall game with you and your students. You can use envelopes instead of doors and could make the price a small piece of candy and then have the other envelopes have small rocks of the same size or shape. You could also use any other sort of prize you feel is appropriate.
[00:15:57.45] CAMERON: The final activity is to have your students think and talk about situations where they can pick the optimal decision but not necessarily yield the optimum outcome.
[00:16:05.78] SAM: Thank you very much for choosing our Blossoms lesson. We hope you and your students have as much fun with it as we did making it.
[00:16:19.96] Now we'll go over the decision tree Cameroon and I made and show you how we used it to determine that switching results in a 2/3 probability of winning the car.
[00:16:31.01] CAMERON: The first step in the decision is the host choice of which door to place the car behind, and there's an equal probability for each of the three possibilities.
[00:16:40.00] SAM: Next comes the contestant's first choice. He or she chooses a door, and we assume, has equal probability of picking any of the three doors. We list what is behind each of the three doors the contestant could select for each of the possible choices the host could have made.
[00:16:58.31] CAMERON: Now comes the most important step, when the assistant reveals the goat. The key thing to notice here is that the assistant only has a choice when the contestant picked the door with the car behind it. If I had picked a door with a goat behind it, then the assistant must open the only other goat door.

[00:17:14.64] SAM: Once the assistant revealed the goat, the contestant is offered the chance to switch. We write down the outcome if he or she switches. Now the last step is to go calculate the probabilities associated with all these different outcomes by multiplying the probabilities corresponding to each step of the decision.
[00:17:34.70] CAMERON: Now that we have the probabilities, we can add up all the probabilities of all the cases where switching the door wins you the car. When we do so, we see they add up to 2/3, and we see that not switching adds up to 1/3 chance of winning.
[00:17:48.51] SAM: So we just used the decision tree to show that 2/3 of the time, switching wins you the car. Something that might help you get a sense for why this is the case is to think about what happens if you initially pick the goat and then switch doors.
[00:18:04.86] CAMERON: You always win because when you switch, you'll always be switching to a car.
[00:18:08.92] SAM: Exactly, and since there are initially three doors and two goats, the probability of initially picking a goat is 2/3. And so if you initially pick a goat, then you need to switch doors to win so that means that 2/3 of the time when you switch doors, you'll win because 2/3 of the time you'll have initially picked a goat. And that's exactly what the decision tree confirms.
[00:18:36.76] CAMERON: A good way to get a better sense of why this is the case is to think about what would happen if we added more doors and more goats.
[00:18:42.04] [MUSIC PLAYING]