Why Pay More? (The Concept of Multiple Regression Analysis)

Assalammualaikum (peace upon you)
Hi everyone
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I teach Property Management Valuation, Mass Appraisal and Property Taxation at the Faculty of Geoinformation and Real Estate.

I am going to talk about Multiple Regression Analysis or in short, MRA. MRA is a mathematical concept which is applicable in many fields when looking at the relationship between one variable - called the dependent variable – with multiple other variables – called independent variables.

MRA is used in numerous activities in our daily lives from simple concepts such as the calculation of our Body Mass Index to the application of intricate calculations such as analyzing the strategies used by big retail stores to calculate factors which could increase their sales.

I will explain Multiple Regression Analysis from the perspective of a property valuer. In property valuation MRA is primarily used to determine the property value, especially in cases where many properties need to be valued, for example, for the purpose of property taxation. MRA is an advanced technique which provides standardized, efficient, uniform, and fair method of property valuation for taxation purposes.
Some important key terms
that you should know
are property value,
building value,
land value,
building area,
factors affecting value,
price, market value.

In most countries,
every property is subject to property tax.
This tax is used by the local authorities
to provide and maintain basic infrastructures,
facilities, and services to the public.
Therefore, if you own a property,
you need to pay property tax.

The question is how is this tax
determined by the local authority? How is it calculated?

Let’s take a look at this situation.

These people seem to have a problem at hand.
Can you help them answer these two basic questions?

What factors determine the property tax?
What factors determine property value?
Discuss the answers with your friends.

In most countries, and especially here in Malaysia,
there are five basic methods of property valuation.

They are the comparison method,
the investment method,
the profit method,
the cost method
and the residual method.

To have a better understanding of these concepts,
please read handout provided by your teacher.

These 5 methods are
considered as traditional methods
of property valuation.
Of all the 5 methods,
the comparison method is the main
and most commonly used in practice.
In property tax valuation,
local authorities are still using
these traditional methods which have been
proven to be too costly and time consuming.
To overcome this problem,
an advanced technique of valuation using MRA
has started to be more widely used
and practiced by assessors.

The basic formula for property valuation
using MRA is basic algebra equation.
Let us consider a simple equation,
an equation which all of you are familiar with.
First equation,
\[2x + 3y = 19\]
equation 2,
\[2x + 5y = 29\]

Now find the solution values of x and y.
Your teacher will assist you
if you have problems.
I will give you 5 minutes to
solve these simple problems.
I'll see you later to discuss the answer.

Do you have your answer ready?
Compare your answers to mine.

Did you and your class also agree on this answer?
If not, you may want to go back
and check your calculations.

Now let me show you a more realistic situation.
Consider we have two houses,
House A
and House B
with the same land area of
22 feet width
and 70 feet depth.
Thus, the land area is equal to 1,540 square feet.
Meanwhile,
the built up area of house A is
22 feet by 45 feet
with two floors.
Thus the built up area is 1,980 square feet.
House B is also 22 feet by 45 feet with two floors. But it has an extended kitchen of 22 feet X 10 feet. Thus the built up area is 2,200 square feet.

The price of House A is RM235,400 and the price of House B is RM253,000. These problems can be expressed in mathematical formulas. For house A, we can write

1,540 L plus 1,980 B equals RM235,400.

For house B, we can write

1,540 L plus 2,200 B equals RM253,000.

What is the value of L and what is the value of B? By using the similar previous process, we can solve these problems.

First, equation (2) minus equation (1). Thus, you’ll get

220 B equals to RM17,600.

Thus, B equals to RM80

Image: B=RM80

Insert B equals to RM80 into the first equation.

1,540 L plus RM158,400 equals to RM235,400.

1,540 L equals to RM235,400 minus RM158,400.

Thus L equals RM50 per square foot.

So, what is the price of:
The land (L) per square foot?
Where L is land upon which the building is erected.

And the built up area (B) per square foot?
Where B is built up area of the house
Now, figure out how much is the price of House C with land area of 1,540 sq. ft.
and a built up area of 2,400 sq. ft.
To solve this problem,
you will need to form the equations based on the information given.

I’ll give you five minutes to solve this problem.

Now, you have the property value based on 2 variables, land and building, you should get $B = 80$ and $L = 50$ and the value of House $C = \text{RM}269,000$.

If you got a different answer, you may want to go back and check your calculations.

Now I understand the basic concept of property valuation.

But these two variables are not enough. Surely there are other factors.

Of course, the other factors or variables in addition to land area and build up area include surrounding facilities and renovation works on the property and many others. Yes, I think this is where we need to use Multiple Regression Analysis.

Yes, Mustafa is correct. We need to use MRA for many multiple variables.

To understand the basic MRA, here, you can see 7 properties. ABCDEF and SP.

SP is the subject property which we want to value based on the prices of 6 comparable properties (ABCDEF). These properties have different features.

Property A features House+Car porch+Pool, the price is 820,000
Property B features House+Car porch, the price is 720,000
Property C features House+pool+Studio, the price is 900,000
Property D features House+Studio+garage, the price is 870,000
Property E features House+Garage+Tennis Court, the price is 930,000
Property F features House+Pool, the price is 780,000

The Subject Property features House+Car porch+Pool+Garage
Given the figure as such,
what is the value of each feature?
Discuss the answer with your friends.

By comparing the prices
of property A and B,
we will get the price of each
utility or component by decomposing
each property into its separate components.
Decomposing here means
separating the components of the property
into its individual price.

For example,
for properties A and B,
we know that property A which is
made up of the house,
carporch, and pool
is priced at 820,000 Malaysian Ringgit.
While property B
which is made up of only
a house and carporch,
is priced at 720,000 Malaysian Ringgit.
Therefore, the price of the pool would be
820,000 – 720,000 = 100,000 Malaysian Ringgit.

By using the same similar process,
we will be able to calculate the price
of the other components of the properties.

Thus, by cleverly using successive substitutions,
the price of the components of the properties
are as follows:
From A and B we will get the price of pool equal to 100,000
From A and F we will get the price of car porch equal to 40,000
From C and F we will get the price of studio equal to 120,000
From A, B and F we will get the price of house equal to 680,000
From A, B, C and D we will get the price of garage equal to 70,000
From 4, 5, and E we will get the price of tennis court equal to 180,000
What I have just demonstrated was
the use of paired comparison method
to calculate the prices of the components
of the properties.
There are three steps in paired comparison method.

Step 1
This approach is used to find,
by deduction, the value attributable
to each major different property feature
exhibited by a target property
in relation to the various sale properties.
The assumption behind this is that
if there is one major difference
in the features like size, facilities, fittings etc,
of two properties which have been sold,
then the difference in price between
the two represents the value
of the single different feature.

Step 2
If there are several differences
then we will need several sales
to be able to isolate the value differences,
and we may have to go through
quite a long process of elimination
in order to eventually account
for all the different value differences.
Assume that we have information
on the following sales,
properties A to F,
each of which is different
from the others in one or more features.
We then carry out
A simple subtractive process
to identify the value of each feature

To arrive at the value
of the subject property,
we need to add up the price
of all components that make up
the subject property.

This is step 3 Subject Property (SP)
What is the value of SP which includes
a house, carporch, pool and garage?
Using the result from the inference process, we can estimate the value of the Subject Property which consists of House, Carporch, Pool and Garage.

**ESTIMATION OF SUBJECT PROPERTY:**

\[ SP = \text{House} + \text{Carporch} + \text{Pool} + \text{Garage} = ? \]
\[ SP = 680,000 + 100,000 + 70,000 + 40,000 \]
\[ SP = \text{RM} 890,000 \]

This means that the more components present in a property, the more it contributes to the property value and this will increase the tax imposed.

Ghazali now understands why his tax is higher than the neighbors. The higher the value, the higher the tax is.

Up to this point we have learned to solve linear equations in order to determine the value of properties. The activities that we have done were simple demonstrations towards the understanding of MRA. In real world case, it involves more complex calculations due to more factors, which need to be taken into consideration in determining property value. It also involves a huge number of properties to be analyzed thus further complicating the inference process.

Due to this, we need to employ MRA which is capable to analyze a huge number of data. A statistical tool like Statistical Package for Social Scientist (SPSS) can be used to run MRA. SPSS makes using MRA much easier, performing the necessary calculations quickly and accurately. However, the real property market is not as simple as that.
Property market is imperfect.
Refer back to the
previous equations from Segment 3

1540 L + 1980 B = RM235,400 ----------(1)
1540 L + 2200 B = RM 253,000----------(2)

In Real property market,
the equations might be as follows

1540 L + 1980 B = RM235,400 ----------(1)
= RM250,000
= RM240,000
= RM260,000
= RM270,000

1540 L + 2200 B = RM 253,000----------(2)
= RM 265,000
= RM 275,000
= RM 280,000
= RM 250,000

Meaning that,
with the same variables,
the price of the house varies between each other,
depends on how much each buyer
is willing to pay for that particular house.

The calculation for multiple variables
requires the use of
Multiple Regression Analysis.

MRA determines the statistical
relationship between Y (property value)
and several independent variables X1, X2,..Xn as follows:

\[ Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + \ldots + b_nX_n \]
This is the regression equation
to be estimated in determining
the property values.
Where Y is the dependent variable
(property value), b0, b1.....bn
are the parameters that are estimated.
X1,X2 ,X3......Xn are independent variables
(all important property characteristics)
which is X1 is Land area,
X2 is Building area,
X3 is Car porch,
X4 Number of bedrooms,
X5 is Ensuite,
X6 is Repairs etc.
Other variables might be Gated,
Guarded, Extension Kitchen,
Extension Car porch,
Site location (Intermediate lot, End lot or Corner lot)
Distance to school,
Distance to Shopping Complex,
Distance to Playing ground and so on.
Variables X1 to Xn may be numeric or categorical.

Property values can be estimated using regression model based on a sample of previously sold properties.
We can predict a value for the dependent variable (property value) based on the known value of the causal (independent) variables, or more critically, MRA also estimates the individual components of value e.g. value of garage, value of car porch and other important components.

The point is that the influence on property value of each independent variables will be estimated by the model, and thus, not subject to human error. As a example, with respect to the Building Area (X2), every additional square feet of the building, will result additional amount in property value (y). This relationship can be easily and accurately measured by using MRA. From the regression result, the estimated value of the subject property can be determined by multiplying each coefficient estimated (b0, b1......bn ) from the equation with the subject property’s respective characteristics. As a result, Regression models produce average, or typical values,
consistent with the definition of market value. So, it is similar to the paired sale analysis and should produce the same result as traditional methods of valuation. There is a great deal of similarity between MRA and traditional methods, when appraisers use a weighted average to arrive a final value estimate they are effectively applying the assumption of the regression to the mean. You can learn more about this concept by reading Handout A.

The aim of this video is to make it easier for tertiary students to understand the concept of Multiple Regression Analysis. It is through our experience that Malaysian tertiary students find this concept a bit difficult to understand. In order to facilitate their understanding, we introduced a scaffolding method whereby students are first exposed to basic equations which they have learned in secondary school. Then, students will be introduced to the concept of variables, teaching them to calculate property value based only 2 variables. Exposing them to multiple variables related to valuation of property further enhances their understanding. Finally, they are asked to calculate property value and property tax based on multiple variables.

To make the video interesting, dramatization is employed to drive home the points. Furthermore, this video will also make heavy use of animations during the professor’s explanation of the topic.

For property valuation students, a prerequisite needed before learning this lesson is the Sales Comparison Method in valuation. For other students such as those in business or social science fields, an understanding of basic statistics
such as total, average, mod, mean, and median is necessary before starting this lesson. The suggested activities are the activities stated in the activity section of this concept and architecture paper.

MRA can also be applied in other areas as well such as medicine. MRA can be used to predict the height of a child which can depend on the height of the mother, father, nutrition, and environmental factors. Social science MRA can be used in many research situations where there will be a lot of independent variables to be considered. For example to analyze the relationship between economic development and its impact to property value. Business MRA can be used by a real estate agent to sell houses. The selling price of a house can depend on the location, the number of bedrooms, the number of bathrooms, the year the house was built, the square footage of the lot, and a number of other factors. There is a limitation in regression analysis. Regression result only gives average value for all the variables. But in real world, it could be difference value. For example, not all square footage in a house is created equal as produces by regression analysis.

So, as with all mathematical techniques applied to real world situations, the user must be careful to know the method's applicability and its limitations. Finally, the professor challenges the students to investigate other possible application of MRA in real life.
Can you think of other possible application of MRA in real life?

Discuss with your friends and your teacher.

BYE-BYE FOR NOW ...

AND GOOD DAY TO ALL OF YOU.

ASSALAMUALAIKUM, WRH. WBTH.